

13. UNA RACCOLTA CONCLUSIVA DI ESERCIZI

(con risposte alla fine della rassegna, seguite dagli svolgimenti completi!)

1) $\lim_{x \rightarrow 0^+} \frac{ae^{\frac{1}{x}} + b}{ce^{\frac{1}{x}} + d}$

2) $\lim_{x \rightarrow 0^-} \frac{ae^{\frac{1}{x}} + b}{ce^{\frac{1}{x}} + d}$

3) $\lim_{x \rightarrow 0} \frac{ax + \operatorname{sen} bx}{cx + \operatorname{sen} dx}$

4) $\lim_{x \rightarrow \infty} \frac{ax + \operatorname{sen} bx}{cx + \operatorname{sen} dx}$

5) $\lim_{x \rightarrow \pm\infty} \left(\frac{3x+4}{5x+6} \right)^x$

6) $\lim_{x \rightarrow 0^+} \frac{e^{\sqrt{x}} - 1}{x}$

7) $\lim_{x \rightarrow 0^+} \frac{\ln x}{\sqrt{x}}$

8) $\lim_{x \rightarrow +\infty} 5^{1-2x}$

9) $\lim_{x \rightarrow +\infty} \ln \frac{x}{x^2 - 1}$

10) $\lim_{x \rightarrow +\infty} \ln \frac{x^2}{x^2 - 1}$

11) $\lim_{x \rightarrow +\infty} \ln \frac{x^3}{x^2 - 1}$

12) $\lim_{x \rightarrow +\infty} e^{\frac{x+2}{x^2-x+1}}$

13) $\lim_{x \rightarrow +\infty} e^{\frac{x^2+2}{x^2-x+1}}$

14) $\lim_{x \rightarrow +\infty} e^{\frac{x^3+2}{x^2-x+1}}$

15) $\lim_{x \rightarrow -\infty} e^{\frac{x^3+2}{x^2-x+1}}$

16) $\lim_{x \rightarrow +\infty} \operatorname{arctg} \frac{x+1}{x^2-x}$

17) $\lim_{x \rightarrow +\infty} \operatorname{arctg} \frac{x^2+1}{x^2-x}$

18) $\lim_{x \rightarrow +\infty} \operatorname{arctg} \frac{x^3+1}{x^2-x}$

19) $\lim_{x \rightarrow -\infty} \operatorname{arctg} \frac{x^3+1}{x^2-x}$

20) $\lim_{x \rightarrow 0} \operatorname{arctg} \frac{x^2+1}{x^2-x}$

21) $\lim_{x \rightarrow +\infty} \left(\frac{x^2}{x^2-1} \right)^x$

22) $\lim_{x \rightarrow +\infty} \left(\frac{x}{x^2-1} \right)^x$

23) $\lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2-1} \right)^x$

24) $\lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 x}{x^3}$

25) $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\pi - x}$

26) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$

27) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{sen}^2 x + \operatorname{sen} x - 2}{\left(x - \frac{\pi}{2} \right)^2}$

28) $\lim_{x \rightarrow +\infty} \frac{x \cdot |x-1|}{2x^2+1}$

29) $\lim_{x \rightarrow -\infty} \frac{x \cdot |x-1|}{2x^2+1}$

30) $\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + b_1 x + c_1} - \sqrt{x^2 + b_2 x + c_2} \right)$

31) $\lim_{x \rightarrow 0} \frac{\ln(1 + \alpha x)}{\ln(1 + \beta x)}$

... E TERMINIAMO QUESTA RASSEGNA
CON DUE ESERCIZI DAVVERO DIFFICILI,
CHE PROPONIAMO IN FORMA "GUIDATA".

$$32) \lim_{x \rightarrow 0^+} x^x \quad [0^0]$$

Applicando la nota identità

$$[f(x)]^{g(x)} = e^{\ln[f(x)]^{g(x)}} = e^{g(x) \cdot \ln f(x)}$$

avremo:

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{\ln x^x} = \lim_{x \rightarrow 0^+} e^{\boxed{x \ln x}}.$$

Ma per quanto riguarda l'esponente:

$$\lim_{x \rightarrow 0^+} (x \ln x) = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{-\ln \frac{1}{x}}{\frac{1}{x}} = - \lim_{x \rightarrow 0^+} \frac{\ln \frac{1}{x}}{\frac{1}{x}} = - \lim_{t \rightarrow +\infty} \frac{\ln t}{t} = 0$$

*perché il logaritmo,
come dimostreremo,
tende all'infinito
meno rapidamente
di ogni funzione algebrica*

da cui

$$\lim_{x \rightarrow 0^+} x^x = \dots$$

$$33) \lim_{x \rightarrow +\infty} \left(\frac{1}{x+1} \right)^{\frac{1}{\ln x}} \quad [0^0]$$

Si ha

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(\frac{1}{x+1} \right)^{\frac{1}{\ln x}} &= \\ &= \lim_{x \rightarrow +\infty} e^{\ln \left[\left(\frac{1}{x+1} \right)^{\frac{1}{\ln x}} \right]} = \lim_{x \rightarrow +\infty} e^{\frac{1}{\ln x} \cdot \ln \frac{1}{x+1}} = \lim_{x \rightarrow +\infty} e^{\frac{1}{\ln x} \cdot [-\ln(x+1)]} = \lim_{x \rightarrow +\infty} e^{\boxed{\frac{\ln(x+1)}{\ln x}}} \end{aligned}$$

Ma è

$$\lim_{x \rightarrow +\infty} \frac{\ln(x+1)}{\ln x} = \lim_{x \rightarrow +\infty} \frac{\ln \left[x \left(1 + \frac{1}{x} \right) \right]}{\ln x} = \lim_{x \rightarrow +\infty} \frac{\ln x + \ln \left(1 + \frac{1}{x} \right)}{\ln x} = \dots$$

da cui

$$\lim_{x \rightarrow +\infty} \left(\frac{1}{x+1} \right)^{\frac{1}{\ln x}} = \dots$$

RISPOSTE

- 1) $\frac{a}{c}$ 2) $\frac{b}{d}$ 3) $\frac{a+b}{c+d}$ 4) $\frac{a}{c}$ 5) $\begin{cases} 0 & \text{con } x \rightarrow +\infty \\ +\infty & \text{con } x \rightarrow -\infty \end{cases}$ 6) $+\infty$ 7) $-\infty$ 8) $\frac{1}{\sqrt{5}}$
- 9) $-\infty$ 10) 0 11) $+\infty$ 12) 1 13) e 14) $+\infty$ 15) 0 16) 0 17) $\frac{\pi}{4}$ 18) $\frac{\pi}{2}$ 19) $-\frac{\pi}{2}$
- 20) $\begin{cases} \lim_{x \rightarrow 0^+} \dots = -\frac{\pi}{2} \\ \lim_{x \rightarrow 0^-} \dots = +\frac{\pi}{2} \end{cases}$ 21) 1 22) 0 23) $+\infty$ 24) ∞ 25) 0 26) 1
- 27) $-\frac{3}{2}$ 28) $\frac{1}{2}$ 29) $-\frac{1}{2}$ 30) $\frac{b_1 - b_2}{2}$ 31) $\frac{\alpha}{\beta}$ 32) 1 33) $\frac{1}{e}$

RISOLUZIONI COMPLETE DEGLI ESERCIZI PRECEDENTI

$$1) \lim_{x \rightarrow 0^+} \frac{\overbrace{ae^{\frac{1}{x}} + b}^{+\infty}}{\underbrace{ce^{\frac{1}{x}} + d}_{+\infty}} = \lim_{x \rightarrow 0^+} \frac{\cancel{e^{\frac{1}{x}}} \left(a + \frac{b}{e^{\frac{1}{x}}} \right)}{\cancel{e^{\frac{1}{x}}} \left(c + \frac{d}{e^{\frac{1}{x}}} \right)} = \frac{a}{c} \quad \text{in quanto } \left. \frac{b}{e^{\frac{1}{x}}} \right\} \rightarrow 0, \left. \frac{d}{e^{\frac{1}{x}}} \right\} \rightarrow 0 \quad 2) \lim_{x \rightarrow 0^-} \frac{\overbrace{ae^{\frac{1}{x}} + b}^0}{\underbrace{ce^{\frac{1}{x}} + d}_0} = \frac{b}{d}$$

$$3) \lim_{x \rightarrow 0} \frac{\overbrace{ax + \text{sen } bx}^0}{\underbrace{cx + \text{sen } dx}_0} = \lim_{x \rightarrow 0} \frac{ax + \text{sen } bx}{cx + \text{sen } dx} = \lim_{x \rightarrow 0} \frac{x}{x} = \lim_{x \rightarrow 0} \frac{a + \frac{\text{sen } bx}{x}}{c + \frac{\text{sen } dx}{x}} \stackrel{\text{NOTA}}{=} \frac{a+b}{c+d}$$

NOTA:

$$\lim_{x \rightarrow 0} \frac{\text{sen } kx}{x} = k,$$

come era già noto:

$$\lim_{x \rightarrow 0} \frac{\text{sen } kx}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{\boxed{\frac{\text{sen } kx}{kx}}}{\boxed{kx}} \cdot k = k$$

↓
sostituzione
implicita $kx=t$

$$4) \lim_{x \rightarrow \infty} \frac{\overbrace{ax + \text{sen } bx}^{\substack{\infty \\ \text{oscilla} \\ \text{fra } -1 \text{ e } 1}}}{\underbrace{cx + \text{sen } dx}_{\substack{\infty \\ \text{oscilla} \\ \text{fra } -1 \text{ e } 1}}} = \lim_{x \rightarrow \infty} \frac{ax + \text{sen } bx}{cx + \text{sen } dx} = \lim_{x \rightarrow \infty} \frac{x}{x} = \lim_{x \rightarrow \infty} \frac{a + \frac{\text{sen } bx}{x}}{c + \frac{\text{sen } dx}{x}} = \frac{a}{c} \quad (\text{NOTA})$$

NOTA: $\lim_{x \rightarrow \infty} \frac{\overbrace{\text{sen } kx}^{\substack{\text{oscilla} \\ \text{fra } -1 \text{ e } 1}}}{\underbrace{x}_{\infty}} = 0.$

Ciò è intuitivamente evidente, ma comunque dimostriamolo:

$$|\text{sen } kx| \leq 1 \Rightarrow \frac{|\text{sen } kx|}{|x|} \leq \frac{1}{|x|};$$

$$\text{quindi } \left| \frac{\text{sen } kx}{x} \right| \leq \frac{1}{|x|}$$

$$\text{e poiché } \lim_{x \rightarrow \infty} \frac{1}{|x|} = 0$$

sarà anche, per il Secondo Teorema del Confronto, $\lim_{x \rightarrow \infty} \frac{\text{sen } kx}{x} = 0$, c.v.d.

$$5) \lim_{x \rightarrow \pm\infty} \left(\frac{\overbrace{3x+4}^{x \rightarrow \pm\infty}}{\underbrace{5x+6}_{\downarrow 3/5}} \right) = \begin{cases} 0 & \text{con } x \rightarrow +\infty \\ +\infty & \text{con } x \rightarrow -\infty \end{cases}$$

Il contenuto della parentesi tende a 3/5

in quanto è ben noto che

il limite, per $x \rightarrow \infty$,

di un quoziente di due polinomi dello stesso grado,

è uguale al rapporto fra i coefficienti dei due termini di grado massimo.

$$6) \lim_{x \rightarrow 0^+} \frac{e^{\sqrt{x}} - 1}{x} = \lim_{x \rightarrow 0^+} \underbrace{\frac{e^{\sqrt{x}} - 1}{\sqrt{x}}}_1 \cdot \underbrace{\frac{1}{\sqrt{x}}}_{+\infty} = +\infty$$

$$7) \lim_{x \rightarrow 0^+} \frac{\overbrace{\ln x}^{-\infty}}{\underbrace{\sqrt{x}}_{0^+}} = -\infty$$

$$8) \lim_{x \rightarrow +\infty} 5^{\frac{x+3}{1-2x}} = 5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$$

Infatti $\lim_{x \rightarrow +\infty} \frac{x+3}{1-2x} = -\frac{1}{2}$.

E' ben noto che il limite, per $x \rightarrow \infty$, di un quoziente di due polinomi dello stesso grado, è uguale al rapporto fra i coefficienti dei due termini di grado massimo

$$9) \lim_{x \rightarrow +\infty} \ln \underbrace{\left[\frac{x}{x^2-1} \right]}_{0^+} = -\infty$$

$$10) \lim_{x \rightarrow +\infty} \ln \underbrace{\left[\frac{x^2}{x^2-1} \right]}_1 = 0$$

$$11) \lim_{x \rightarrow +\infty} \ln \underbrace{\left[\frac{x^3}{x^2-1} \right]}_{+\infty} = +\infty$$

$$12) \lim_{x \rightarrow +\infty} e^{\left. \frac{x+2}{x^2-x+1} \right\} \rightarrow 0} = 1$$

$$13) \lim_{x \rightarrow +\infty} e^{\left. \frac{x^2+2}{x^2-x+1} \right\} \rightarrow 1} = e$$

$$14) \lim_{x \rightarrow +\infty} e^{\left. \frac{x^3+2}{x^2-x+1} \right\} \rightarrow +\infty} = +\infty$$

$$15) \lim_{x \rightarrow -\infty} e^{\left. \frac{x^3+2}{x^2-x+1} \right\} \rightarrow -\infty} = 0$$

$$16) \lim_{x \rightarrow +\infty} \operatorname{arctg} \underbrace{\left[\frac{x+1}{x^2-x} \right]}_{0^+} = 0$$

$$17) \lim_{x \rightarrow +\infty} \operatorname{arctg} \underbrace{\left[\frac{x^2+1}{x^2-x} \right]}_1 = \frac{\pi}{4}$$

$$18) \lim_{x \rightarrow +\infty} \operatorname{arctg} \underbrace{\left[\frac{x^3+1}{x^2-x} \right]}_{+\infty} = \frac{\pi}{2}$$

$$19) \lim_{x \rightarrow -\infty} \operatorname{arctg} \underbrace{\left[\frac{x^3+1}{x^2-x} \right]}_{-\infty} = -\frac{\pi}{2}$$

$$20) \lim_{x \rightarrow 0} \operatorname{arctg} \underbrace{\left[\frac{x^2+1}{x^2-x} \right]}_0 = \begin{cases} \lim_{x \rightarrow 0^+} \operatorname{arctg} \underbrace{\left[\frac{x^2+1}{x(x-1)} \right]}_{-\infty} = -\frac{\pi}{2} \\ \lim_{x \rightarrow 0^-} \operatorname{arctg} \underbrace{\left[\frac{x^2+1}{x(x-1)} \right]}_{+\infty} = +\frac{\pi}{2} \end{cases}$$

$$21) \lim_{x \rightarrow +\infty} \left(\underbrace{\left[\frac{x^2}{x^2-1} \right]}_1 \right)^{x \rightarrow +\infty} \stackrel{[1^\infty]}{=} \lim_{x \rightarrow +\infty} \left(\frac{x^2-1+1}{x^2-1} \right)^x = \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x^2-1} \right)^x =$$

$$= \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x^2-1} \right)^{(x^2-1) \cdot \frac{x}{x^2-1}} = \lim_{x \rightarrow +\infty} \underbrace{\left[\left(1 + \frac{1}{x^2-1} \right)^{x^2-1} \right]}_e^{\frac{x}{x^2-1} \rightarrow 0} = 1$$

$$22) \lim_{x \rightarrow +\infty} \left(\underbrace{\left[\frac{x}{x^2-1} \right]}_{0^+} \right)^{x \rightarrow +\infty} = 0$$

$$23) \lim_{x \rightarrow +\infty} \left(\underbrace{\left[\frac{x^3}{x^2-1} \right]}_{+\infty} \right)^{x \rightarrow +\infty} = +\infty$$

$$24) \lim_{x \rightarrow 0} \frac{\overbrace{\text{tg}^2 x}^0}{\underbrace{x^3}_0} = \lim_{x \rightarrow 0} \frac{\text{sen}^2 x}{x^3} = \lim_{x \rightarrow 0} \frac{\text{sen}^2 x}{x^3} \cdot \frac{1}{\cos^2 x} = \lim_{x \rightarrow 0} \frac{\text{sen}^2 x}{x^2} \cdot \frac{1}{x} \cdot \frac{1}{\cos^2 x} = \lim_{x \rightarrow 0} \underbrace{\left(\frac{\text{sen} x}{x}\right)^2}_1 \cdot \underbrace{\frac{1}{x}}_{\infty} \cdot \underbrace{\frac{1}{\cos^2 x}}_1 = \infty$$

Per SOSTITUZIONE DI VARIABILE.

Poniamo $\pi - x = t$; avremo $x = \pi - t$, con $t \rightarrow 0$:

$$25) \lim_{x \rightarrow \pi} \frac{\overbrace{1 + \cos x}^0}{\underbrace{\pi - x}_0} = \lim_{x \rightarrow \pi} \frac{1 + \cos x}{\pi - x} = \lim_{t \rightarrow 0} \frac{1 + \cos(\pi - t)}{t} =$$

$$= \lim_{t \rightarrow 0} \frac{1 - \cos t}{t} = \lim_{t \rightarrow 0} \frac{\overbrace{1 - \cos t}^0}{\underbrace{t^2}_{\rightarrow 1/2}} \cdot \underbrace{t}_0 = 0$$

$$26) \lim_{x \rightarrow 0} \frac{\overbrace{\text{arctg} x}^0}{\underbrace{x}_0} = \lim_{t \rightarrow 0} \frac{t}{\text{tg} t} = \lim_{t \rightarrow 0} \frac{1}{\frac{\text{tg} t}{t}} = 1$$

$$27) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\overbrace{\text{sen}^2 x + \text{sen} x - 2}^0}{\underbrace{\left(x - \frac{\pi}{2}\right)^2}_0} = \lim_{t \rightarrow 0} \frac{\text{sen}^2\left(t + \frac{\pi}{2}\right) + \text{sen}\left(t + \frac{\pi}{2}\right) - 2}{t^2} =$$

$$= \lim_{t \rightarrow 0} \frac{\cos^2 t + \cos t - 2}{t^2} = \lim_{t \rightarrow 0} \frac{(\cos t + 2)(\cos t - 1)}{t^2} = - \lim_{t \rightarrow 0} \frac{\overbrace{1 - \cos t}^0}{\underbrace{t^2}_{1/2}} \cdot \underbrace{(\cos t + 2)}_3 = -\frac{3}{2}$$

$$28) \lim_{x \rightarrow +\infty} \frac{\overbrace{x \cdot |x-1|}^{+\infty}}{\underbrace{2x^2+1}^{+\infty}} = \lim_{x \rightarrow +\infty} \frac{x(x-1)}{2x^2+1} = \lim_{x \rightarrow +\infty} \frac{x^2-x}{2x^2+1} = \frac{1}{2}$$

$$29) \lim_{x \rightarrow -\infty} \frac{\overbrace{x \cdot |x-1|}^{-\infty}}{\underbrace{2x^2+1}^{+\infty}} = \lim_{x \rightarrow -\infty} \frac{x(1-x)}{2x^2+1} = \lim_{x \rightarrow -\infty} \frac{x-x^2}{2x^2+1} = -\frac{1}{2}$$

$$30) \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + b_1 x + c_1} - \sqrt{x^2 + b_2 x + c_2} \right) = \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + b_1 x + c_1} - \sqrt{x^2 + b_2 x + c_2} \right) \cdot \frac{\sqrt{x^2 + b_1 x + c_1} + \sqrt{x^2 + b_2 x + c_2}}{\sqrt{x^2 + b_1 x + c_1} + \sqrt{x^2 + b_2 x + c_2}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 + b_1 x + c_1 - x^2 - b_2 x - c_2}{\sqrt{x^2 + b_1 x + c_1} + \sqrt{x^2 + b_2 x + c_2}} = \lim_{x \rightarrow +\infty} \frac{(b_1 - b_2)x + (c_1 - c_2)}{\sqrt{x^2 + b_1 x + c_1} + \sqrt{x^2 + b_2 x + c_2}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left[(b_1 - b_2) + \frac{c_1 - c_2}{x} \right]}{\sqrt{x^2 \left(1 + \frac{b_1}{x} + \frac{c_1}{x^2} \right)} + \sqrt{x^2 \left(1 + \frac{b_2}{x} + \frac{c_2}{x^2} \right)}} = \lim_{x \rightarrow +\infty} \frac{x \left[(b_1 - b_2) + \frac{c_1 - c_2}{x} \right]}{|x| \sqrt{1 + \frac{b_1}{x} + \frac{c_1}{x^2}} + |x| \sqrt{1 + \frac{b_2}{x} + \frac{c_2}{x^2}}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left[(b_1 - b_2) + \frac{c_1 - c_2}{x} \right]}{\cancel{x} \left(\sqrt{1 + \frac{b_1}{x} + \frac{c_1}{x^2}} + \sqrt{1 + \frac{b_2}{x} + \frac{c_2}{x^2}} \right)} = \frac{b_1 - b_2}{2}$$

$$31) \lim_{x \rightarrow 0} \frac{\overbrace{\ln(1+\alpha x)}^0}{\underbrace{\ln(1+\beta x)}_0} = \lim_{x \rightarrow 0} \frac{\ln(1+\alpha x)}{\frac{x}{x}} = \lim_{x \rightarrow 0} \frac{\overbrace{\frac{\ln(1+\alpha x)}{\alpha x}}^1 \cdot \alpha}{\underbrace{\frac{\ln(1+\beta x)}{\beta x}}_1 \cdot \beta} = \frac{\alpha}{\beta}$$

$$32) \lim_{x \rightarrow 0^+} x^x \text{ F.I. } [0^0].$$

Applicando la nota identità

$$[f(x)]^{g(x)} = e^{\ln[f(x)]^{g(x)}} = e^{g(x) \cdot \ln f(x)}$$

avremo:

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{\ln x^x} = \lim_{x \rightarrow 0^+} e^{\boxed{x \ln x}}.$$

Ma per quanto riguarda l'esponente:

$$\lim_{x \rightarrow 0^+} (x \ln x) = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{-\frac{1}{x}}{\frac{1}{x}} = - \lim_{x \rightarrow 0^+} \frac{\ln \frac{1}{x}}{\frac{1}{x}} = - \lim_{t \rightarrow +\infty} \frac{\ln t}{t} = 0$$

*perché il logaritmo,
come dimostreremo,
tende all'infinito
meno rapidamente
di ogni funzione algebrica*

da cui

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{\overbrace{x \ln x}^0} = 1$$

$$33) \lim_{x \rightarrow +\infty} \left(\frac{\boxed{\frac{1}{x+1}}}{\underbrace{\phantom{\frac{1}{x+1}}}_{0^+}} \right)^{\overbrace{\frac{1}{\ln x}}^{\rightarrow 0^+}} \rightarrow 0^+ [0^0] =$$

$$= \lim_{x \rightarrow +\infty} e^{\ln \left[\left(\frac{1}{x+1} \right)^{\frac{1}{\ln x}} \right]} = \lim_{x \rightarrow +\infty} e^{\frac{1}{\ln x} \cdot \ln \frac{1}{x+1}} = \lim_{x \rightarrow +\infty} e^{\frac{1}{\ln x} [-\ln(x+1)]} = \lim_{x \rightarrow +\infty} e^{\frac{\ln(x+1)}{\ln x}}$$

Ma è

$$\lim_{x \rightarrow +\infty} \frac{\ln(x+1)}{\ln x} = \lim_{x \rightarrow +\infty} \frac{\ln \left[x \left(1 + \frac{1}{x} \right) \right]}{\ln x} = \lim_{x \rightarrow +\infty} \frac{\ln x + \ln \left(1 + \frac{1}{x} \right)}{\ln x} = \lim_{x \rightarrow +\infty} \left[1 + \frac{\ln \left(1 + \frac{1}{x} \right)}{\ln x} \right] = 1$$

da cui

$$\lim_{x \rightarrow +\infty} \left(\frac{1}{x+1} \right)^{\frac{1}{\ln x}} = \lim_{x \rightarrow +\infty} e^{\frac{\ln(x+1)}{\ln x}} = e^{-1} = \frac{1}{e}$$