

16. EQUAZIONI E SISTEMI DI PRIMO GRADO CON COEFFICIENTI IRRAZIONALI

Ecco qualche esempio.

$$1) \quad \boxed{x\sqrt{3} - (\sqrt{3} + 1) = x\sqrt{27} - 1}$$

$$x\sqrt{3} - \sqrt{3} \cancel{1} = 3x\sqrt{3} \cancel{1} \quad (\text{OSSERVAZIONE})$$

$$x\sqrt{3} - 3x\sqrt{3} = \sqrt{3}$$

$$-2x\sqrt{3} = \sqrt{3}$$

$$x = \frac{\cancel{\sqrt{3}}}{-2\cancel{\sqrt{3}}} = \boxed{-\frac{1}{2}} \quad \text{Anche: } x\cancel{\sqrt{3}} - 3x\cancel{\sqrt{3}} = \sqrt{3}^1; \quad -2x = 1; \quad x = -\frac{1}{2}$$

OSSERVAZIONE

Si preferisce scrivere
 $x\sqrt{3}$ al posto di $\sqrt{3} \cdot x$
 $3x\sqrt{3}$ anziché $3\sqrt{3} \cdot x$

$$2) \quad \boxed{\frac{x}{\sqrt{5}} - \frac{6}{\sqrt{5}-1} = \frac{x(\sqrt{2}-1)}{5-\sqrt{5}}}$$

$$\frac{x}{\sqrt{5}} - \frac{6}{\sqrt{5}-1} = \frac{x(\sqrt{2}-1)}{\sqrt{5}(\sqrt{5}-1)}$$

$$\frac{(\sqrt{5}-1)x - 6\sqrt{5}}{\cancel{\sqrt{5}(\sqrt{5}-1)}} = \frac{x(\sqrt{2}-1)}{\cancel{\sqrt{5}(\sqrt{5}-1)}}$$

$$x\sqrt{5} \cancel{x} - 6\sqrt{5} = x\sqrt{2} \cancel{x}$$

$$x\sqrt{5} - x\sqrt{2} = 6\sqrt{5}$$

$$x(\sqrt{5} - \sqrt{2}) = 6\sqrt{5}$$

$$x = \frac{6\sqrt{5}}{\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{6\sqrt{5}(\sqrt{5}+\sqrt{2})}{5-2} = \frac{\cancel{6}\sqrt{5}(\sqrt{5}+\sqrt{2})}{\cancel{3}} = \boxed{10+2\sqrt{10}}$$

$$3) \quad \boxed{(\sqrt{15} - \sqrt{5} + 2)x = \frac{3(2x+1)}{\sqrt{3}} - 1}$$

$$(\sqrt{15} - \sqrt{5} + 2)x = \frac{\cancel{\sqrt{3}} \cdot \sqrt{3}(2x+1)}{\cancel{\sqrt{3}}} - 1$$

$$x\sqrt{15} - x\sqrt{5} + 2x = 2x\sqrt{3} + \sqrt{3} - 1$$

$$x\sqrt{15} - x\sqrt{5} + 2x - 2x\sqrt{3} = \sqrt{3} - 1$$

$$x(\sqrt{15} - \sqrt{5} + 2 - 2\sqrt{3}) = \sqrt{3} - 1$$

$$x = \frac{\sqrt{3}-1}{\sqrt{15}-\sqrt{5}+2-2\sqrt{3}} =$$

$$= \frac{\sqrt{3}-1}{\sqrt{5}(\sqrt{3}-1)-2(\sqrt{3}-1)} = \frac{\cancel{\sqrt{3}-1}}{(\cancel{\sqrt{3}-1})(\sqrt{5}-2)} =$$

$$= \frac{1}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \boxed{\sqrt{5}+2}$$

$$4) \quad \boxed{\begin{cases} x\sqrt{3} - y\sqrt{3} = 6 \\ \sqrt{3} \cdot \frac{x+y-2}{y-x} = 1 \end{cases}}$$

$$\begin{cases} x\sqrt{3} - y\sqrt{3} = 6 \\ x\sqrt{3} + y\sqrt{3} - 2\sqrt{3} = y - x \quad (y \neq x) \end{cases}$$

$$\begin{cases} x\sqrt{3} - y\sqrt{3} = 6 \\ x\sqrt{3} + x + y\sqrt{3} - y = 2\sqrt{3} \end{cases}$$

$$\begin{cases} x\sqrt{3} - y\sqrt{3} = 6 \\ x(\sqrt{3}+1) + y(\sqrt{3}-1) = 2\sqrt{3} \end{cases}$$

$$\begin{cases} x\sqrt{3} = y\sqrt{3} + 6 \\ x(\sqrt{3}+1) + y(\sqrt{3}-1) = 2\sqrt{3} \end{cases}$$

$$\begin{cases} x = \frac{y\sqrt{3}+6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3y+6\sqrt{3}}{3} = \frac{\cancel{3}(y+2\sqrt{3})}{\cancel{3}} = y+2\sqrt{3} \\ (y+2\sqrt{3})(\sqrt{3}+1) + y(\sqrt{3}-1) = 2\sqrt{3} \end{cases}$$

$$\begin{cases} x = y+2\sqrt{3} \\ y\sqrt{3} + \cancel{y} + 6 + \cancel{2\sqrt{3}} + y\sqrt{3} - \cancel{y} = 2\sqrt{3} \end{cases}$$

$$\begin{cases} x = y+2\sqrt{3} \\ \cancel{2}y\sqrt{3} = -\cancel{6}^3; \quad y = -\frac{3}{\sqrt{3}} = -\frac{\cancel{\sqrt{3}} \cdot \sqrt{3}}{\cancel{\sqrt{3}}} = -\sqrt{3} \end{cases}$$

$$\begin{cases} x = y+2\sqrt{3} = -\sqrt{3}+2\sqrt{3} = \sqrt{3} \\ y = -\sqrt{3} \end{cases} \quad \boxed{\begin{cases} x = \sqrt{3} \\ y = -\sqrt{3} \end{cases}}$$

ESERCIZI		SOLUZIONI	
5) $x\sqrt{5}+15=0$	6) $3x=2-x\sqrt{7}$	5) $-3\sqrt{5}$	6) $3-\sqrt{7}$
7) $x=2(2+\sqrt{2})-x\sqrt{2}$	8) $\sqrt{3}(x-\sqrt{3})=\sqrt{6}$	7) $2\sqrt{2}$	8) $\sqrt{2}+\sqrt{3}$
9) $3x\sqrt{2}=2+x\sqrt{8}$	10) $x(x+\sqrt{2})=x(x+1)+1$	9) $\sqrt{2}$	10) $\sqrt{2}+1$
11) $x(\sqrt{3}-1)^2=2(2x-\sqrt{6})$	12) $\frac{x\sqrt{5}-1}{\sqrt{2}}=\sqrt{2}-x$	11) $\sqrt{2}$	12) $\sqrt{5}-\sqrt{2}$
13) $\sqrt{2}(x-1)+1=\sqrt{3}(\sqrt{3}+\sqrt{2})$	14) $\frac{x}{\sqrt{3}-1}-\frac{x-1}{\sqrt{3}+1}=\frac{1}{2}$	13) $\sqrt{3}+\sqrt{2}+1$	14) $\frac{2-\sqrt{3}}{2}$
15) $\begin{cases} x\sqrt{5}+y=6 \\ x-y\sqrt{5}=0 \end{cases}$	16) $\begin{cases} x\sqrt{3}+y\sqrt{2}=1 \\ x\sqrt{6}+3y=0 \end{cases}$	15) $\begin{cases} x=\sqrt{5} \\ y=1 \end{cases}$	16) $\begin{cases} x=\sqrt{3} \\ y=-\sqrt{2} \end{cases}$
17) $\begin{cases} x\sqrt{7}-y\sqrt{2}=5 \\ x\sqrt{2}-y\sqrt{7}=0 \end{cases}$	18) $\begin{cases} x\sqrt{5}+y\sqrt{5}=10 \\ x-y=2 \end{cases}$	17) $\begin{cases} x=\sqrt{7} \\ y=\sqrt{2} \end{cases}$	18) $\begin{cases} x=\sqrt{5}+1 \\ y=\sqrt{5}-1 \end{cases}$
19) $\begin{cases} x\sqrt{3}+y\sqrt{2}=4 \\ x\sqrt{3}-y\sqrt{2}=2 \end{cases}$	20) $\begin{cases} (x+y-\sqrt{2})\sqrt{2}+y=x \\ \frac{2+y}{x}=1 \end{cases}$	19) $\begin{cases} x=\sqrt{3} \\ y=\frac{\sqrt{2}}{2} \end{cases}$	20) $\begin{cases} x=\sqrt{2}+1 \\ y=\sqrt{2}-1 \end{cases}$