13. UNA RACCOLTA CONCLUSIVA DI ESERCIZI
(con risposte alla fine della rassegna, seguite dagli svolgimenti completi!)

1) $\lim_{x\to 0^+} \frac{ae^{\frac{1}{x}}+b}{ce^{\frac{1}{x}}+d}$ 2) $\lim_{x\to 0^-} \frac{ae^{\frac{1}{x}}+b}{ce^{\frac{1}{x}}+d}$

3) $\lim_{x\to 0} \frac{ax+sen\,bx}{cx+sen\,dx}$ 4) $\lim_{x\to \infty} \frac{ax+sen\,bx}{cx+sen\,dx}$

5) $\lim_{x\to \pm\infty} \left(\frac{3x+4}{5x+6}\right)^x$

6) $\lim_{x\to 0^+} \frac{e^{\sqrt{x}}-1}{x}$

7) $\underbrace{\lim}_{x\to 0^+} \frac{\ln x}{\sqrt{x}}$

8) $\lim_{x\to+\infty} 5^{\frac{x+3}{1-2x}}$

9) $\lim_{x\to +\infty} \ln\frac{x}{x^2-1}$ 10) $\lim_{x\to +\infty} \ln\frac{x^2}{x^2-1}$ 11) $\lim_{x\to +\infty} \ln\frac{x^3}{x^2-1}$

12) $\lim_{x\to +\infty} e^{\frac{x+2}{x^2-x+1}}$ 13) $\lim_{x\to +\infty} e^{\frac{x^2+2}{x^2-x+1}}$ 14) $\lim_{x\to +\infty} e^{\frac{x^3+2}{x^2-x+1}}$ 15) $\lim_{x\to -\infty} e^{\frac{x^3+2}{x^2-x+1}}$

16) $\lim_{x\to +\infty} arctg\frac{x+1}{x^2-x}$ 17) $\lim_{x\to +\infty} arctg\frac{x^2+1}{x^2-x}$

18) $\lim_{x\to +\infty} arctg\frac{x^3+1}{x^2-x}$ 19) $\lim_{x\to -\infty} arctg\frac{x^3+1}{x^2-x}$ 20) $\lim_{x\to 0} arctg\frac{x^2+1}{x^2-x}$

21) $\lim_{x\to +\infty} \left(\frac{x^2}{x^2-1}\right)^x$ 22) $\lim_{x\to +\infty} \left(\frac{x}{x^2-1}\right)^x$ 23) $\lim_{x\to +\infty} \left(\frac{x^3}{x^2-1}\right)^x$

24) $\lim_{x\to 0} \frac{tg^2 x}{x^3}$

25) $\lim_{x\to \pi} \frac{1+\cos x}{\pi-x}$

26) $\lim_{x\to 0} \frac{arctg\,x}{x}$

27) $\lim_{x\to \frac{\pi}{2}} \frac{sen^2 x+sen x-2}{\left(x-\frac{\pi}{2}\right)^2}$

28) $\lim_{x\to +\infty} \frac{x\cdot|x-1|}{2x^2+1}$ 29) $\lim_{x\to -\infty} \frac{x\cdot|x-1|}{2x^2+1}$

30) $\lim_{x\to +\infty} \left(\sqrt{x^2+b_1x+c_1}-\sqrt{x^2+b_2x+c_2}\right)$

31) $\lim_{x\to 0} \frac{\ln(1+\alpha x)}{\ln(1+\beta x)}$

… E TERMINIAMO QUESTA RASSEGNA
CON DUE ESERCIZI DAVVERO DIFFICILI,
CHE PROPONIAMO IN FORMA "GUIDATA".

32) $\lim_{x\to 0^+} x^x \quad \left[0^0\right]$

Applicando la nota identità

$$[f(x)]^{g(x)} = e^{\ln[f(x)]^{g(x)}} = e^{g(x)\cdot \ln f(x)}$$

avremo:

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\ln x^x} = \lim_{x\to 0^+} e^{\boxed{x\ln x}}.$$

Ma per quanto riguarda l'esponente:

$$\lim_{x\to 0^+}(x\ln x) = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+}\frac{-\ln\frac{1}{x}}{\frac{1}{x}} = -\lim_{x\to 0^+}\frac{\ln\frac{1}{x}}{\frac{1}{x}} = -\lim_{t\to+\infty}\frac{\ln t}{t} = 0$$

perché il logaritmo, come dimostreremo, tende all'infinito meno rapidamente di ogni funzione algebrica

da cui

$$\lim_{x\to 0^+} x^x = \ldots$$

33) $\lim_{x\to+\infty}\left(\frac{1}{x+1}\right)^{\frac{1}{\ln x}} \quad \left[0^0\right]$

Si ha

$$\lim_{x\to+\infty}\left(\frac{1}{x+1}\right)^{\frac{1}{\ln x}} \overset{\left[0^0\right]}{=}$$

$$= \lim_{x\to+\infty} e^{\ln\left[\left(\frac{1}{x+1}\right)^{\frac{1}{\ln x}}\right]} = \lim_{x\to+\infty} e^{\frac{1}{\ln x}\cdot\ln\frac{1}{x+1}} = \lim_{x\to+\infty} e^{\frac{1}{\ln x}\cdot[-\ln(x+1)]} = \lim_{x\to+\infty} e^{\boxed{-\frac{\ln(x+1)}{\ln x}}}$$

Ma è

$$\lim_{x\to+\infty}\frac{\ln(x+1)}{\ln x} = \lim_{x\to+\infty}\frac{\ln\left[x\left(1+\frac{1}{x}\right)\right]}{\ln x} = \lim_{x\to+\infty}\frac{\ln x + \ln\left(1+\frac{1}{x}\right)}{\ln x} = \ldots$$

da cui

$$\lim_{x\to+\infty}\left(\frac{1}{x+1}\right)^{\frac{1}{\ln x}} = \ldots$$

RISPOSTE

1) $\frac{a}{c}$ 2) $\frac{b}{d}$ 3) $\frac{a+b}{c+d}$ 4) $\frac{a}{c}$ 5) $\begin{cases} 0 & con\ x\to+\infty \\ +\infty & con\ x\to-\infty \end{cases}$ 6) $+\infty$ 7) $-\infty$ 8) $\frac{1}{\sqrt{5}}$

9) $-\infty$ 10) 0 11) $+\infty$ 12) 1 13) e 14) $+\infty$ 15) 0 16) 0 17) $\frac{\pi}{4}$ 18) $\frac{\pi}{2}$ 19) $-\frac{\pi}{2}$

20) $\begin{cases} \lim_{x\to 0^+}\ldots = -\frac{\pi}{2} \\ \lim_{x\to 0^-}\ldots = +\frac{\pi}{2} \end{cases}$ 21) 1 22) 0 23) $+\infty$ 24) ∞ 25) 0 26) 1

27) $-\frac{3}{2}$ 28) $\frac{1}{2}$ 29) $-\frac{1}{2}$ 30) $\frac{b_1-b_2}{2}$ 31) $\frac{\alpha}{\beta}$ 32) 1 33) $\frac{1}{e}$

RISOLUZIONI COMPLETE DEGLI ESERCIZI PRECEDENTI

1) $$\lim_{x\to 0^+}\frac{\overset{+\infty}{\overbrace{a e^{\frac{1}{x}}}}+b}{\underset{+\infty}{\underbrace{c e^{\frac{1}{x}}}}+d}=\lim_{x\to 0^+}\frac{\cancel{e^{\frac{1}{x}}}\left(a+\dfrac{b}{e^{\frac{1}{x}}}\right)}{\cancel{e^{\frac{1}{x}}}\left(c+\dfrac{d}{e^{\frac{1}{x}}}\right)}=\frac{a}{c}\quad \text{in quanto}\quad \left.\frac{b}{e^{\frac{1}{x}}}\right\}\to 0,\quad \left.\frac{d}{e^{\frac{1}{x}}}\right\}\to 0$$

2) $$\lim_{x\to 0^-}\frac{\overset{0}{\overbrace{a e^{\frac{1}{x}}}}+b}{\underset{0}{\underbrace{c e^{\frac{1}{x}}}}+d}=\frac{b}{d}$$

3) $$\lim_{x\to 0}\frac{\overset{0}{\overbrace{ax+\operatorname{sen}bx}}}{\underset{0}{\underbrace{cx+\operatorname{sen}dx}}}=\lim_{x\to 0}\frac{\dfrac{ax+\operatorname{sen}bx}{x}}{\dfrac{cx+\operatorname{sen}dx}{x}}=\lim_{x\to 0}\frac{a+\overset{b}{\overbrace{\boxed{\dfrac{\operatorname{sen}bx}{x}}}}}{c+\underset{d}{\underbrace{\boxed{\dfrac{\operatorname{sen}dx}{x}}}}}\overset{\text{NOTA}}{=}\frac{a+b}{c+d}$$

NOTA:

$$\lim_{x\to 0}\frac{\operatorname{sen}kx}{x}=k\,,$$

come era già noto:

$$\lim_{x\to 0}\frac{\operatorname{sen}kx}{x}=$$

$$=\lim_{x\to 0}\underset{1}{\underbrace{\boxed{\frac{\operatorname{sen}kx}{kx}}}}\cdot k=k$$

sostituzione implicita kx=t

4) $$\lim_{x\to\infty}\frac{\overset{\infty}{\overbrace{ax}}+\overset{\text{oscilla fra }-1\text{ e }1}{\operatorname{sen}bx}}{\underset{\infty}{\underbrace{cx}}+\underset{\text{oscilla fra }-1\text{ e }1}{\operatorname{sen}dx}}=\lim_{x\to\infty}\frac{\dfrac{ax+\operatorname{sen}bx}{x}}{\dfrac{cx+\operatorname{sen}dx}{x}}=\lim_{x\to\infty}\frac{a+\overset{0}{\overbrace{\boxed{\dfrac{\operatorname{sen}bx}{x}}}}}{c+\underset{0}{\underbrace{\boxed{\dfrac{\operatorname{sen}dx}{x}}}}}=\frac{a}{c}\quad\text{(NOTA)}$$

NOTA: $$\lim_{x\to\infty}\frac{\overset{\text{oscilla fra }-1\text{ e }1}{\operatorname{sen}kx}}{\underset{\infty}{\underbrace{x}}}=0.$$

Ciò è intuitivamente evidente, ma comunque dimostriamolo:

$$|\operatorname{sen}kx|\le 1\Rightarrow\frac{|\operatorname{sen}kx|}{|x|}\le\frac{1}{|x|}\,;$$

quindi $$\left|\frac{\operatorname{sen}kx}{x}\right|\le\left|\frac{1}{x}\right|$$

e poiché $$\lim_{x\to\infty}\left|\frac{1}{x}\right|=0$$

sarà anche, per il Secondo Teorema del Confronto, $\displaystyle\lim_{x\to\infty}\frac{\operatorname{sen}kx}{x}=0$, c.v.d.

5) $$\lim_{x\to\pm\infty}\left(\underset{3/5}{\underbrace{\boxed{\frac{3x+4}{5x+6}}}}\right)^{x\}\to\pm\infty}=\begin{cases}0 & con\ x\to+\infty\\ +\infty & con\ x\to-\infty\end{cases}$$

Il contenuto della parentesi tende a 3/5
in quanto è ben noto che
il limite, per $x\to\infty$,
di un quoziente di due polinomi dello stesso grado,
è uguale al rapporto fra i coefficienti dei due termini di grado massimo.

6) $\lim\limits_{x\to 0^+} \dfrac{e^{\sqrt{x}}-1}{x} = \lim\limits_{x\to 0^+} \underbrace{\boxed{\dfrac{e^{\sqrt{x}}-1}{\sqrt{x}}}}_{\substack{\downarrow\\ 1}} \cdot \underbrace{\boxed{\dfrac{1}{\sqrt{x}}}}_{\substack{\downarrow\\ +\infty}} = +\infty$

7) $\underbrace{\lim}_{x\to 0^+} \dfrac{\overbrace{\ln x}^{\substack{-\infty\\ \uparrow}}}{\underbrace{\sqrt{x}}_{\substack{\downarrow\\ 0+}}} = -\infty$

8) $\lim\limits_{x\to+\infty} 5^{\frac{x+3}{1-2x}} = 5^{-\frac{1}{2}} = \dfrac{1}{\sqrt{5}}$

Infatti $\lim\limits_{x\to+\infty} \dfrac{x+3}{1-2x} = -\dfrac{1}{2}$.
E' ben noto che
il limite, per $x\to\infty$, di un quoziente di due polinomi dello stesso grado,
è uguale al rapporto fra i coefficienti dei due termini di grado massimo

9) $\lim\limits_{x\to+\infty} \ln \underbrace{\boxed{\dfrac{x}{x^2-1}}}_{\substack{\downarrow\\ 0^+}} = -\infty$

10) $\lim\limits_{x\to+\infty} \ln \underbrace{\boxed{\dfrac{x^2}{x^2-1}}}_{\substack{\downarrow\\ 1}} = 0$

11) $\lim\limits_{x\to+\infty} \ln \underbrace{\boxed{\dfrac{x^3}{x^2-1}}}_{\substack{\downarrow\\ +\infty}} = +\infty$

12) $\lim\limits_{x\to+\infty} e^{\left.\frac{x+2}{x^2-x+1}\right\}\to 0} = 1$

13) $\lim\limits_{x\to+\infty} e^{\left.\frac{x^2+2}{x^2-x+1}\right\}\to 1} = e$

14) $\lim\limits_{x\to+\infty} e^{\left.\frac{x^3+2}{x^2-x+1}\right\}\to +\infty} = +\infty$

15) $\lim\limits_{x\to-\infty} e^{\left.\frac{x^3+2}{x^2-x+1}\right\}\to -\infty} = 0$

16) $\lim\limits_{x\to+\infty} arctg \underbrace{\boxed{\dfrac{x+1}{x^2-x}}}_{\substack{\downarrow\\ 0^+}} = 0$

17) $\lim\limits_{x\to+\infty} arctg \underbrace{\boxed{\dfrac{x^2+1}{x^2-x}}}_{\substack{\downarrow\\ 1}} = \dfrac{\pi}{4}$

18) $\lim\limits_{x\to+\infty} arctg \underbrace{\boxed{\dfrac{x^3+1}{x^2-x}}}_{\substack{\downarrow\\ +\infty}} = \dfrac{\pi}{2}$

19) $\lim\limits_{x\to-\infty} arctg \underbrace{\boxed{\dfrac{x^3+1}{x^2-x}}}_{\substack{\downarrow\\ -\infty}} = -\dfrac{\pi}{2}$

20) $\lim\limits_{x\to 0} arctg \dfrac{\overbrace{x^2+1}^{\substack{1\\ \uparrow}}}{\underbrace{x^2-x}_{\substack{\downarrow\\ 0}}} =$

$\lim\limits_{x\to 0^+} arctg \underbrace{\boxed{\dfrac{\overbrace{x^2+1}^{\substack{1\\ \uparrow}}}{\underbrace{x}_{\substack{\downarrow\\ 0^+}}\underbrace{(x-1)}_{\substack{\downarrow\\ -1}}}}}_{\substack{\downarrow\\ -\infty}} = -\dfrac{\pi}{2}$

$\lim\limits_{x\to 0^-} arctg \underbrace{\boxed{\dfrac{\overbrace{x^2+1}^{\substack{1\\ \uparrow}}}{\underbrace{x}_{\substack{\downarrow\\ 0^-}}\underbrace{(x-1)}_{\substack{\downarrow\\ -1}}}}}_{\substack{\downarrow\\ +\infty}} = +\dfrac{\pi}{2}$

21) $\lim\limits_{x\to+\infty} \left(\underbrace{\boxed{\dfrac{x^2}{x^2-1}}}_{\substack{\downarrow\\ 1}}\right)^{x\,\}\to+\infty} \overset{[1^\infty]}{=} \lim\limits_{x\to+\infty}\left(\dfrac{x^2-1+1}{x^2-1}\right)^x = \lim\limits_{x\to+\infty}\left(1+\dfrac{1}{x^2-1}\right)^x =$

$= \lim\limits_{x\to+\infty}\left(1+\dfrac{1}{x^2-1}\right)^{(x^2-1)\cdot\frac{x}{x^2-1}} = \lim\limits_{x\to+\infty}\left[\underbrace{\boxed{\left(1+\dfrac{1}{x^2-1}\right)^{x^2-1}}}_{\substack{\downarrow\\ e}}\right]^{\left.\frac{x}{x^2-1}\right\}\to 0} = 1$

22) $\lim\limits_{x\to+\infty} \left(\underbrace{\boxed{\dfrac{x}{x^2-1}}}_{\substack{\downarrow\\ 0^+}}\right)^{x\,\}\to+\infty} = 0$

23) $\lim\limits_{x\to+\infty} \left(\underbrace{\boxed{\dfrac{x^3}{x^2-1}}}_{\substack{\downarrow\\ +\infty}}\right)^{x\,\}\to+\infty} = +\infty$

24) $$\lim_{x\to 0}\frac{\overbrace{tg^2 x}^{\to 0}}{\underbrace{x^3}_{\to 0}} = \lim_{x\to 0}\frac{\frac{sen^2 x}{cos^2 x}}{x^3} = \lim_{x\to 0}\frac{sen^2 x}{x^3}\cdot\frac{1}{cos^2 x} = \lim_{x\to 0}\frac{sen^2 x}{x^2}\cdot\frac{1}{x}\cdot\frac{1}{cos^2 x} = \lim_{x\to 0}\underbrace{\boxed{\left(\frac{sen\, x}{x}\right)^2}}_{\to 1}\cdot\underbrace{\frac{1}{x}}_{\to\infty}\cdot\underbrace{\frac{1}{cos^2 x}}_{\to 1} = \infty$$

25) $$\lim_{x\to\pi}\frac{\overbrace{1+cos\, x}^{\to 0}}{\underbrace{\pi - x}_{\to 0}}$$

Per SOSTITUZIONE DI VARIABILE.

Poniamo $\pi - x = t$; avremo $x = \pi - t$, con $t \to 0$:

$$\lim_{x\to\pi}\frac{1+cos\, x}{\pi - x} = \lim_{t\to 0}\frac{1+cos(\pi - t)}{t} =$$

$$= \lim_{t\to 0}\frac{1-cos\, t}{t} = \lim_{t\to 0}\underbrace{\boxed{\frac{1-cos\, t}{t^2}}}_{\to 1/2}\cdot\underbrace{t}_{\to 0} = 0$$

26) $$\lim_{x\to 0}\frac{\overbrace{arctg\, x}^{\to 0}}{\underbrace{x}_{\to 0}} \overset{\substack{arctg\, x = t\\ tg\, t = x\\ x\to 0\\ t\to 0}}{=} \lim_{t\to 0}\frac{t}{tg\, t} = \lim_{t\to 0}\frac{1}{\left.\frac{tg\, t}{t}\right\}\to 1} = 1$$

27) $$\lim_{x\to\frac{\pi}{2}}\frac{\overbrace{sen^2 x + sen x - 2}^{\to 0}}{\underbrace{\left(x-\frac{\pi}{2}\right)^2}_{\to 0}} \overset{\substack{x-\frac{\pi}{2}=t\\ x=t+\frac{\pi}{2}\\ x\to\frac{\pi}{2}\\ t\to 0}}{=} \lim_{t\to 0}\frac{sen^2\left(t+\frac{\pi}{2}\right)+sen\left(t+\frac{\pi}{2}\right)-2}{t^2} =$$

$$= \lim_{t\to 0}\frac{cos^2 t + cos\, t - 2}{t^2} = \lim_{t\to 0}\frac{(cos\, t+2)(cos\, t-1)}{t^2} = -\lim_{t\to 0}\underbrace{\boxed{\frac{1-cos\, t}{t^2}}}_{\to 1/2}\cdot\underbrace{(cos\, t+2)}_{\to 3} = -\frac{3}{2}$$

28) $$\lim_{x\to+\infty}\frac{\overbrace{x\cdot|x-1|}^{\to+\infty}}{\underbrace{2x^2+1}_{\to+\infty}} = \lim_{x\to+\infty}\frac{x(x-1)}{2x^2+1} = \lim_{x\to+\infty}\frac{x^2-x}{2x^2+1} = \frac{1}{2}$$

29) $$\lim_{x\to-\infty}\frac{\overbrace{x\cdot|x-1|}^{\to-\infty}}{\underbrace{2x^2+1}_{\to+\infty}} = \lim_{x\to-\infty}\frac{x(1-x)}{2x^2+1} = \lim_{x\to-\infty}\frac{x-x^2}{2x^2+1} = -\frac{1}{2}$$

30) $$\lim_{x\to+\infty}\left(\sqrt{x^2+b_1x+c_1}-\sqrt{x^2+b_2x+c_2}\right) = \lim_{x\to+\infty}\left(\sqrt{x^2+b_1x+c_1}-\sqrt{x^2+b_2x+c_2}\right)\cdot\frac{\sqrt{x^2+b_1x+c_1}+\sqrt{x^2+b_2x+c_2}}{\sqrt{x^2+b_1x+c_1}+\sqrt{x^2+b_2x+c_2}} =$$

$$= \lim_{x\to+\infty}\frac{x^2+b_1x+c_1-x^2-b_2x-c_2}{\sqrt{x^2+b_1x+c_1}+\sqrt{x^2+b_2x+c_2}} = \lim_{x\to+\infty}\frac{(b_1-b_2)x+(c_1-c_2)}{\sqrt{x^2+b_1x+c_1}+\sqrt{x^2+b_2x+c_2}} =$$

$$= \lim_{x\to+\infty}\frac{x\left[(b_1-b_2)+\frac{c_1-c_2}{x}\right]}{\sqrt{x^2\left(1+\frac{b_1}{x}+\frac{c_1}{x^2}\right)}+\sqrt{x^2\left(1+\frac{b_2}{x}+\frac{c_2}{x^2}\right)}} = \lim_{x\to+\infty}\frac{x\left[(b_1-b_2)+\frac{c_1-c_2}{x}\right]}{|x|\sqrt{1+\frac{b_1}{x}+\frac{c_1}{x^2}}+|x|\sqrt{1+\frac{b_2}{x}+\frac{c_2}{x^2}}} =$$

$$= \lim_{x\to+\infty}\frac{x\left[(b_1-b_2)+\frac{c_1-c_2}{x}\right]}{x\sqrt{1+\frac{b_1}{x}+\frac{c_1}{x^2}}+x\sqrt{1+\frac{b_2}{x}+\frac{c_2}{x^2}}} = \lim_{x\to+\infty}\frac{\cancel{x}\left[(b_1-b_2)+\frac{c_1-c_2}{x}\right]}{\cancel{x}\left(\sqrt{1+\frac{b_1}{x}+\frac{c_1}{x^2}}+\sqrt{1+\frac{b_2}{x}+\frac{c_2}{x^2}}\right)} = \frac{b_1-b_2}{2}$$

31) $$\lim_{x\to 0}\frac{\overbrace{\ln(1+\alpha x)}^{\to 0}}{\underbrace{\ln(1+\beta x)}_{\to 0}}=\lim_{x\to 0}\frac{\dfrac{\ln(1+\alpha x)}{x}}{\dfrac{\ln(1+\beta x)}{x}}=\lim_{x\to 0}\frac{\overbrace{\boxed{\dfrac{\ln(1+\alpha x)}{\alpha x}}}^{\to 1}\cdot\alpha}{\underbrace{\boxed{\dfrac{\ln(1+\beta x)}{\beta x}}}_{\to 1}\cdot\beta}=\frac{\alpha}{\beta}$$

32) $\lim_{x\to 0^+} x^x \quad F.I.\left[0^0\right].$

Applicando la nota identità

$$[f(x)]^{g(x)}=e^{\ln\left[f(x)\right]^{g(x)}}=e^{g(x)\cdot\ln f(x)}$$

avremo:

$$\lim_{x\to 0^+} x^x=\lim_{x\to 0^+} e^{\ln x^x}=\lim_{x\to 0^+} e^{\boxed{x\ln x}}.$$

Ma per quanto riguarda l'esponente:

$$\lim_{x\to 0^+}(x\ln x)=\lim_{x\to 0^+}\frac{\ln x}{\dfrac{1}{x}}=\lim_{x\to 0^+}\frac{-\ln\dfrac{1}{x}}{\dfrac{1}{x}}=-\lim_{x\to 0^+}\frac{\ln\dfrac{1}{x}}{\dfrac{1}{x}}=-\lim_{t\to +\infty}\frac{\ln t}{t}=0$$

perché il logaritmo, come dimostreremo, tende all'infinito meno rapidamente di ogni funzione algebrica

da cui

$$\lim_{x\to 0^+} x^x=\lim_{x\to 0^+} e^{\overbrace{x\ln x}^{\to 0}}=1$$

33) $$\lim_{x\to +\infty}\left(\underbrace{\boxed{\frac{1}{x+1}}}_{\to 0^+}\right)^{\left.\frac{1}{\ln x}\right\}\to 0^+}\overset{\left[0^0\right]}{=}$$

$$=\lim_{x\to +\infty} e^{\ln\left[\left(\frac{1}{x+1}\right)^{\frac{1}{\ln x}}\right]}=\lim_{x\to +\infty} e^{\frac{1}{\ln x}\cdot\ln\frac{1}{x+1}}=\lim_{x\to +\infty} e^{\frac{1}{\ln x}\cdot\left[-\ln(x+1)\right]}=\lim_{x\to +\infty} e^{\boxed{-\frac{\ln(x+1)}{\ln x}}}$$

Ma è

$$\lim_{x\to +\infty}\frac{\ln(x+1)}{\ln x}=\lim_{x\to +\infty}\frac{\ln\left[x\left(1+\dfrac{1}{x}\right)\right]}{\ln x}=\lim_{x\to +\infty}\frac{\ln x+\ln\left(1+\dfrac{1}{x}\right)}{\ln x}=\lim_{x\to +\infty}\left[1+\underbrace{\boxed{\frac{\ln\left(1+\dfrac{1}{x}\right)}{\ln x}}}_{\to 0}\right]=1$$

da cui

$$\lim_{x\to +\infty}\left(\frac{1}{x+1}\right)^{\frac{1}{\ln x}}=\lim_{x\to +\infty} e^{\boxed{-\frac{\ln(x+1)}{\ln x}}}=e^{-1}=\frac{1}{e}$$