

ESERCIZI**Studia le seguenti funzioni e tracciane il grafico:**

1) $y = 3x^2 - x^3$

2) $y = x^4 - 6x^2$

3) $y = \frac{1}{16}(x-3)^2(x+3)$

4) $y = \frac{x^2 - 3}{x - 2}$

5) $y = \frac{x^3 - 8}{4x}$

6) $y = \frac{4}{x^2 + 2x - 3}$

7) $y = \frac{3x^2 - 3}{x^3}$

8) $y = \frac{x^3}{3x^2 - 3}$

9) $y = \frac{x}{x^3 + 1}$

10) $y = \left(\frac{x}{x-2}\right)^2$

11) $y = \frac{x^2}{(x+1)(x+2)}$

12) $y = \sqrt{x}(x-2)$

13) $y = \frac{\sqrt{x}}{x-2}$

14) $y = \frac{x-4}{\sqrt{x^2+1}}$

15) $y = x\sqrt{\frac{x-1}{x}}$

16) $y = \frac{x}{\sqrt{x-4}}$

17) $y = \sqrt{x^2+1} + 2x$

18) $y = \sqrt{x^2-1} + 2x$

19) $y = \sqrt[3]{8-x^3}$

20) $y = \sqrt[3]{4-x^2}$

21) $y = \frac{x}{\sqrt[3]{x^2 - 1}}$

22) $y = xe^x$

23) $y = e^{x-x^2}$

24) $y = \frac{e^{x^2}}{x^2}$

25) $y = (1+x^2)e^x$

26) $y = e^{2x} - e^x$

27) $y = \frac{\ln x}{\sqrt{x}}$

28) $y = \frac{x}{\ln x}$

29) $y = \ln^2 x - \ln x$

30) $y = \ln(1+e^x)$

31) $y = \frac{\ln x + 1}{\ln x - 1}$

32) $y = \operatorname{sen} x + \cos x \quad su [0, 2\pi]$

33) $y = \cos x(1 - \cos x) \quad su [0, 2\pi]$

34) $y = \operatorname{sen}^2 x \cos x \quad su [0, 2\pi]$

35) $y = \frac{2\operatorname{sen} x - 1}{2\operatorname{sen} x + 1} \quad su [0, 2\pi]$

36) $y = \operatorname{sen} x + x$

37) $y = \frac{\operatorname{sen} x}{\operatorname{sen} x + \cos x} \quad su [0, 2\pi]$

38) $y = e^{\operatorname{arctg} x}$

39) $y = x \operatorname{arctg} x$

40) $y = x - 2 \operatorname{arctg} x$

41) $y = \frac{|x-2|-1}{x^2}$

42) $y = \frac{x|x+2|}{2(|x|-3)}$

RISPOSTE

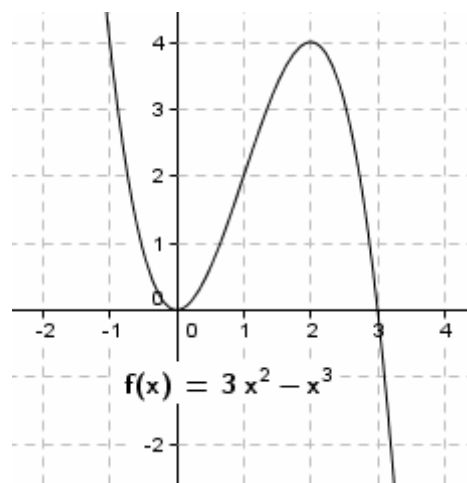
1)

$$y = 3x^2 - x^3$$

$$\text{min}(0,0);$$

$$\text{Max}(2,4);$$

$$\text{Flex}(1,2)$$



2)

$$y = x^4 - 6x^2$$

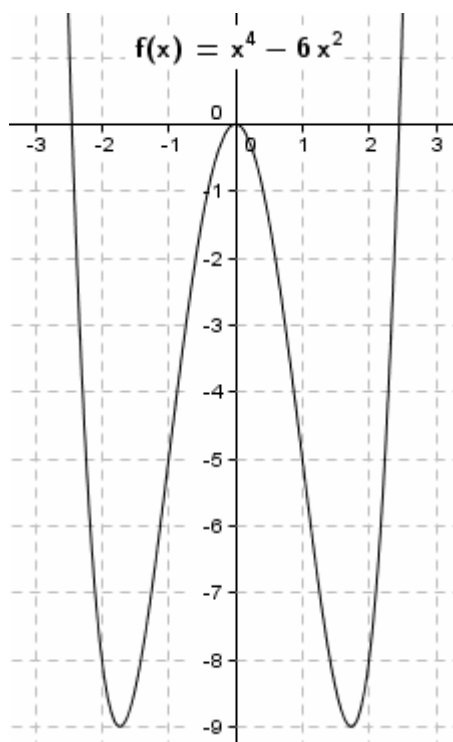
$$\text{min}_1(-\sqrt{3}, -9);$$

$$\text{Max}(0,0);$$

$$\text{min}_2(\sqrt{3}, -9);$$

$$\text{Flex}_1(-1, -5);$$

$$\text{Flex}_2(1, -5)$$



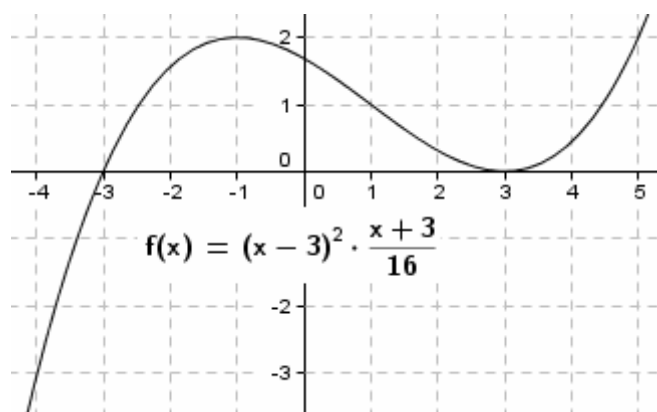
3)

$$y = \frac{1}{16}(x-3)^2(x+3)$$

$$\text{Max}(-1,2);$$

$$\text{min}(3,0);$$

$$\text{Flex}(1,1)$$



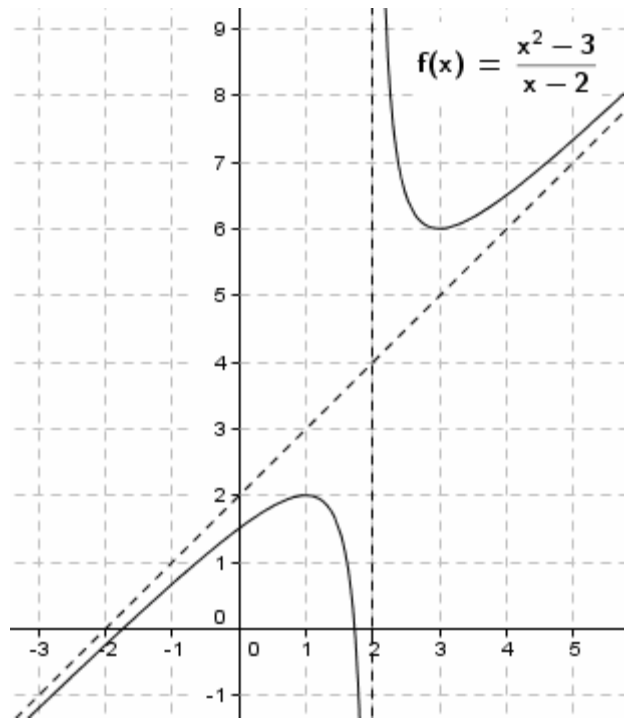
4)

$$y = \frac{x^2 - 3}{x - 2}$$

$Max(1, 2);$

$min(3, 6);$

$asintoti\ x = 2, y = x + 2$



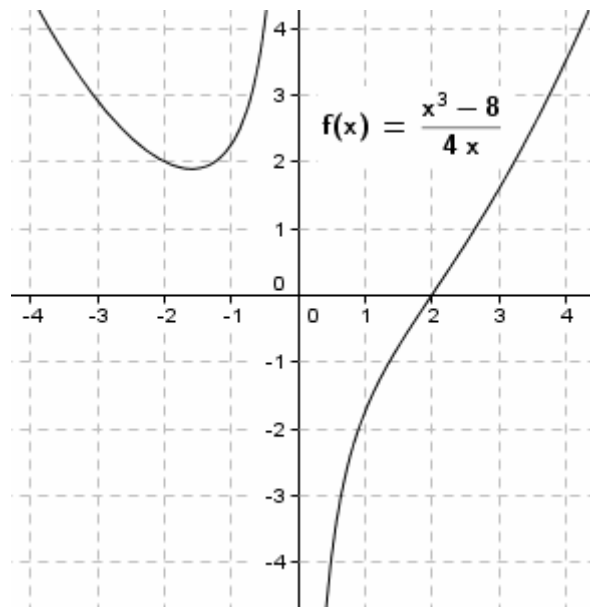
5)

$$y = \frac{x^3 - 8}{4x}$$

$min_1\left(-\sqrt[3]{4}, \frac{3}{\sqrt[3]{4}}\right);$

$Flex(2, 0);$

$asintoto : x = 0$

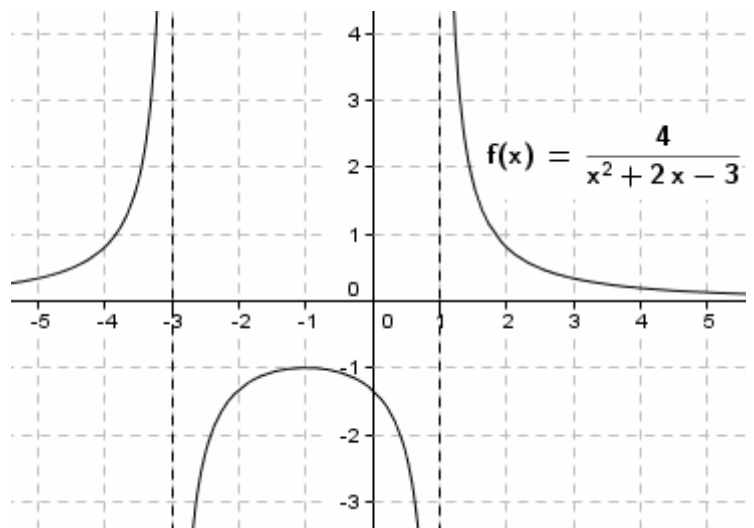


6)

$$y = \frac{4}{x^2 + 2x - 3}$$

$Max(-1, -1);$

$asintoti : x = -3, x = 1, y = 0$



7)

$$y = \frac{3x^2 - 3}{x^3}$$

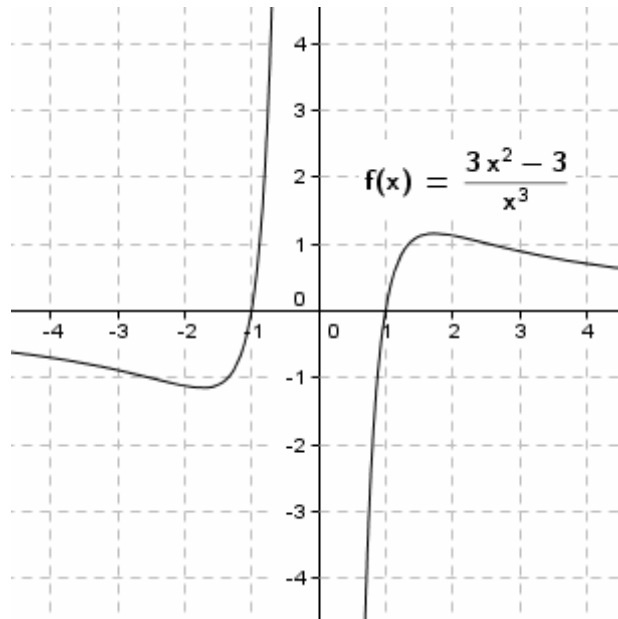
$$\min\left(-\sqrt{3}, -\frac{2}{\sqrt{3}}\right);$$

$$\text{Max}\left(\sqrt{3}, \frac{2}{\sqrt{3}}\right);$$

$$\text{Flex}_1\left(-\sqrt{6}, -\frac{5}{2\sqrt{6}}\right);$$

$$\text{Flex}_2\left(\sqrt{6}, \frac{5}{2\sqrt{6}}\right);$$

$$\text{asintoti: } x=0, y=0$$



8)

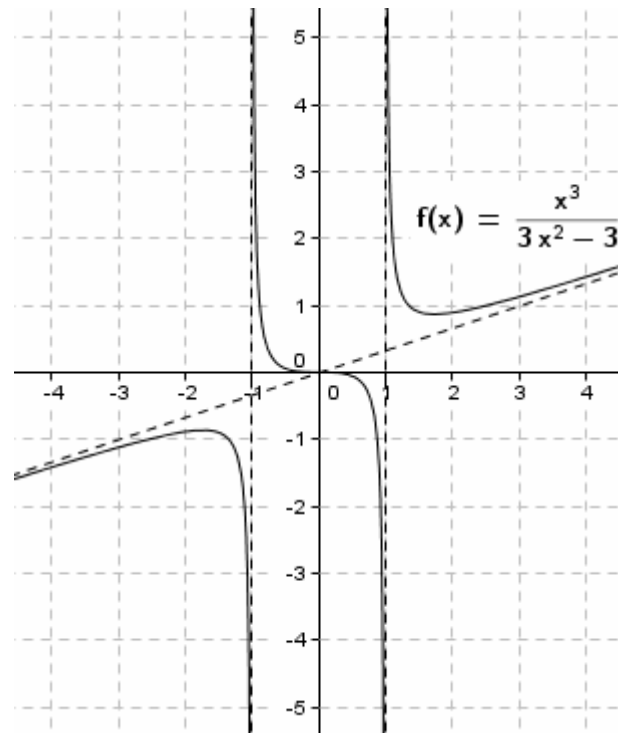
$$y = \frac{x^3}{3x^2 - 3}$$

$$\text{Max}\left(-\sqrt{3}, -\frac{\sqrt{3}}{2}\right);$$

$$\min\left(\sqrt{3}, \frac{\sqrt{3}}{2}\right);$$

$$\text{Flex}(0, 0);$$

$$\text{asintoti: } x = -1, x = 1, y = \frac{1}{3}x$$



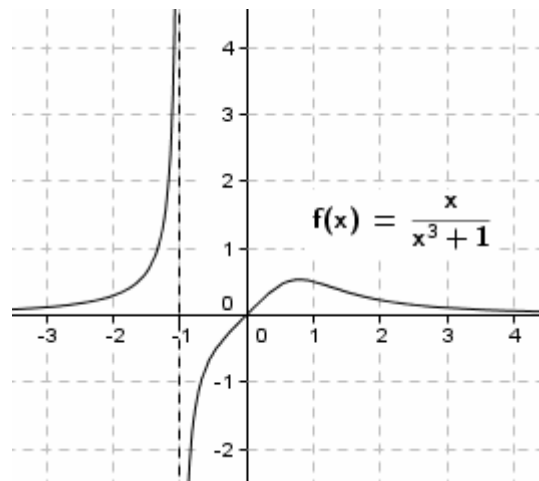
9)

$$y = \frac{x}{x^3 + 1}$$

$$\text{Max}\left(\frac{1}{\sqrt[3]{2}}, \frac{\sqrt[3]{4}}{3}\right);$$

$$\text{Flex}\left(\sqrt[3]{2}, \frac{\sqrt[3]{2}}{3}\right);$$

$$\text{asintoti: } x = -1, y = 0$$



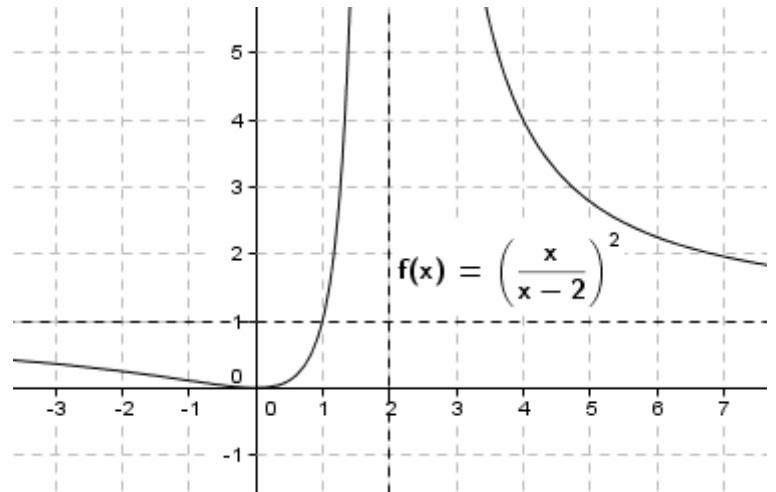
10)

$$y = \left(\frac{x}{x-2} \right)^2$$

$$\min(0, 0);$$

$$\text{Flex} \left(-1, \frac{1}{9} \right);$$

$$\text{asintoti} : x = 2, y = 1$$



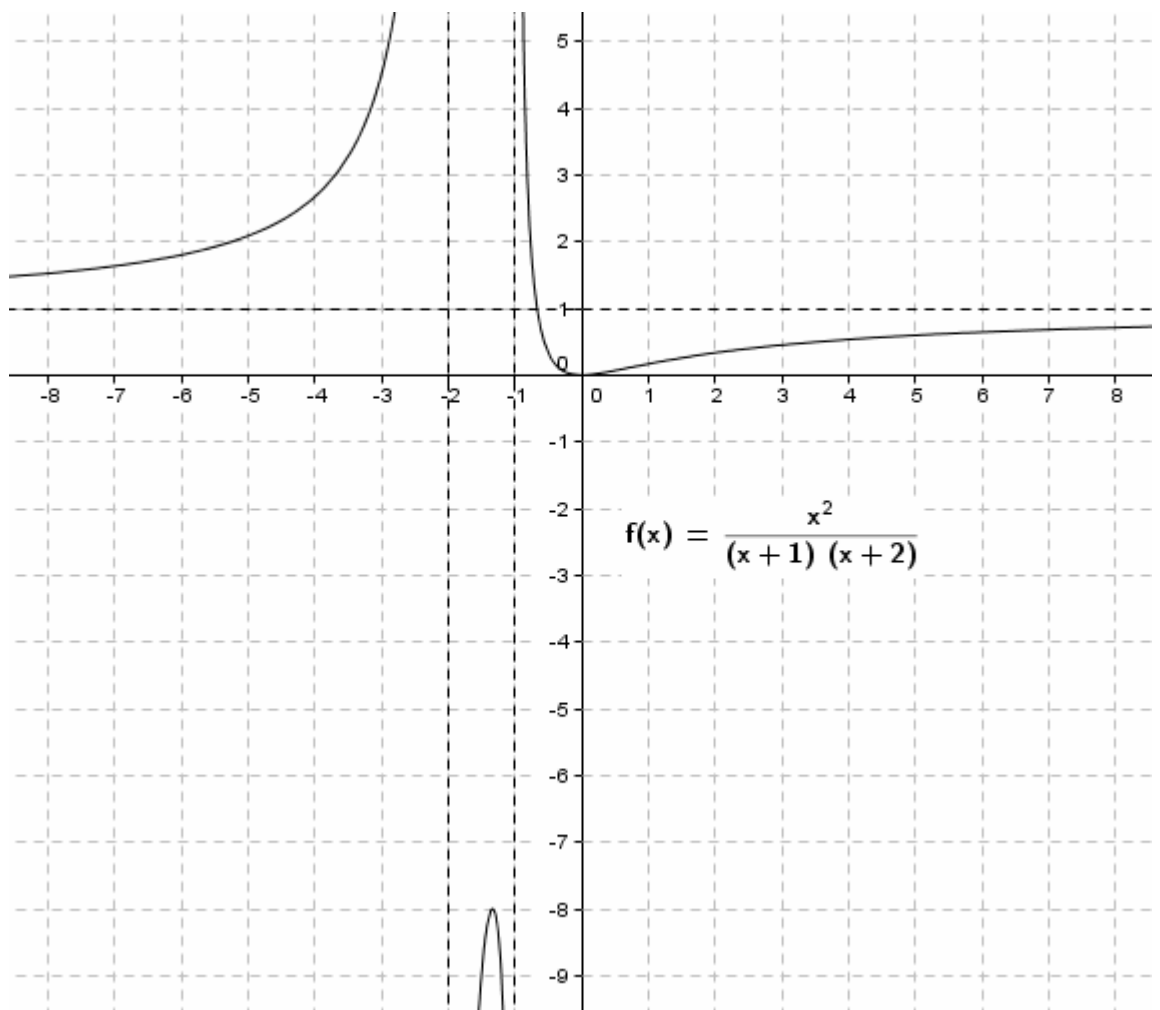
11)

$$y = \frac{x^2}{(x+1)(x+2)}$$

$$\text{Max} \left(-\frac{4}{3}, -8 \right); \min(0, 0); \text{asintoti} : x = -2, x = -1, y = 1$$

C'è un flesso, la cui ascissa vale circa 0.7; lo si può localizzare risolvendo graficamente l'equazione

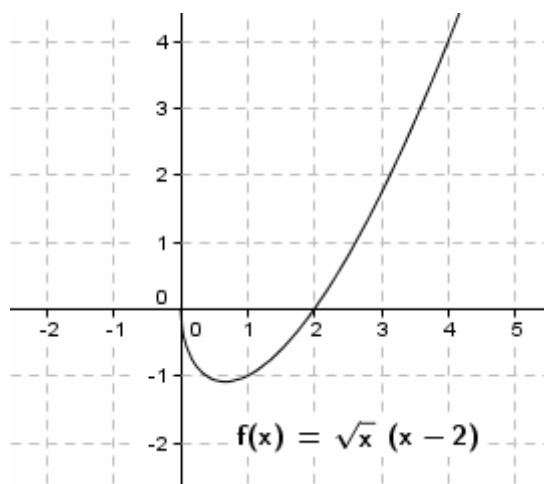
$$3x^3 + 6x^2 - 4 = 0 \text{ ossia } x^3 = -2x^2 + \frac{4}{3}$$



12)

$$y = \sqrt{x}(x-2)$$

$$\min\left(\frac{2}{3}, -\frac{4\sqrt{6}}{9}\right)$$

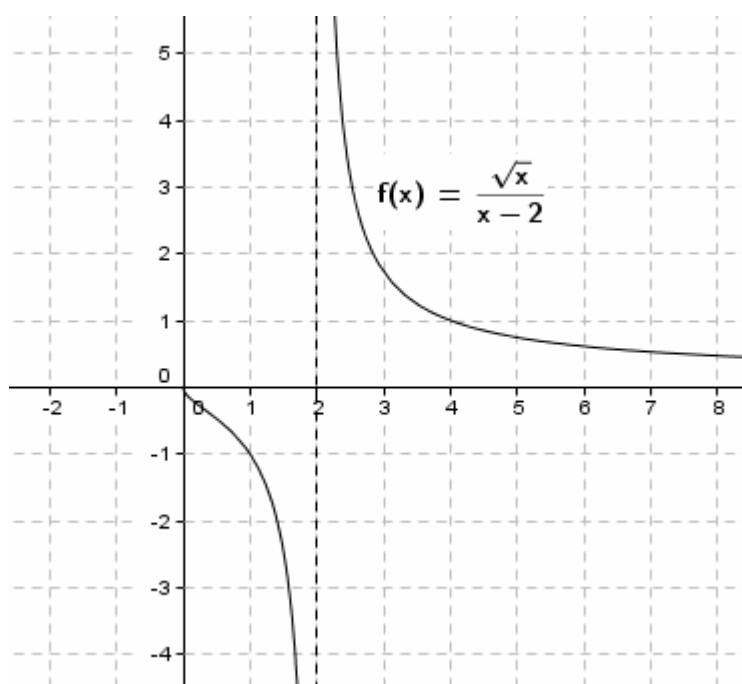


13)

$$y = \frac{\sqrt{x}}{x-2}$$

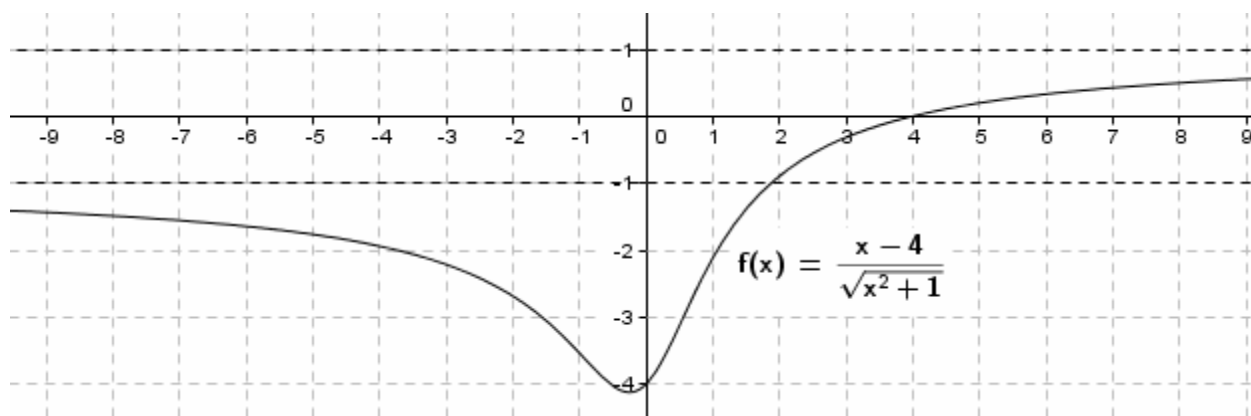
Flesso di ascissa $\frac{-6+4\sqrt{3}}{3}$;

asintoti : $x=2, y=0$



14)

$$y = \frac{x-4}{\sqrt{x^2+1}} \quad \min\left(-\frac{1}{4}, -\sqrt{17}\right); \quad \text{asintoti} : y = \pm 1; \quad \text{due flessi di ascisse } \frac{-3 \pm \sqrt{137}}{16}$$

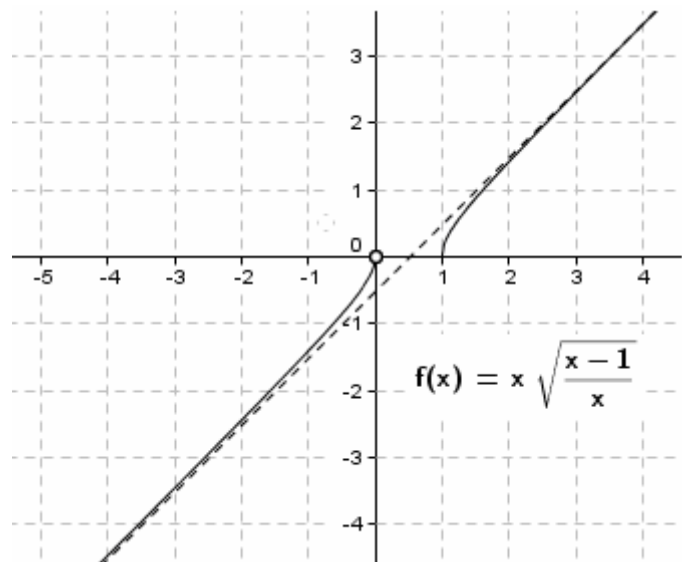


15)

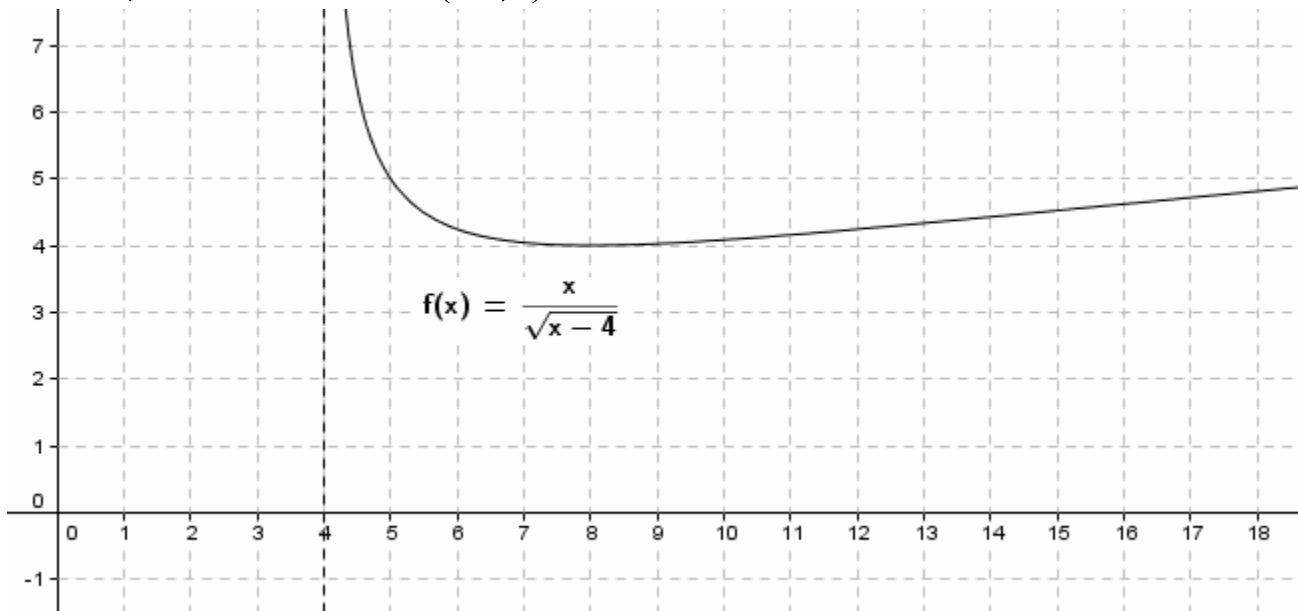
$$y = x \sqrt{\frac{x-1}{x}}$$

$$\text{Asintoto: } y = x - \frac{1}{2}$$

$$\lim_{x \rightarrow 0^-} f(x) = 0^-$$



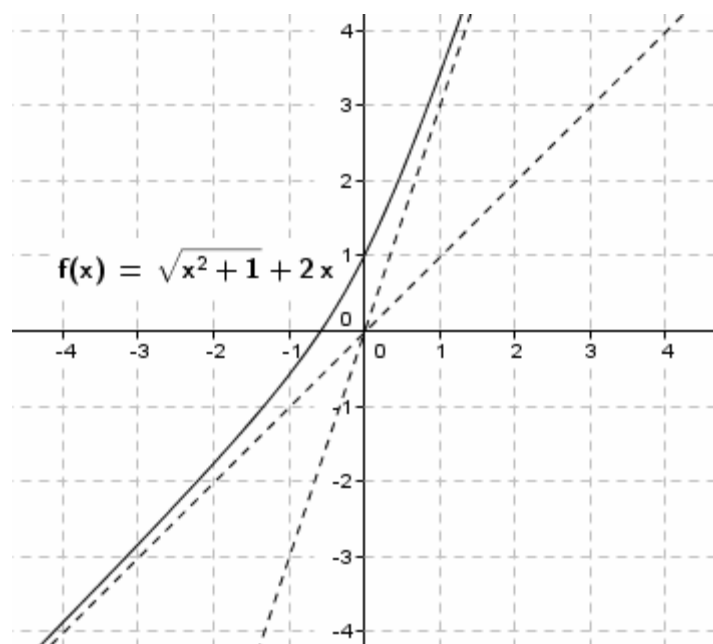
16) $y = \frac{x}{\sqrt{x-4}}$ $\min(8,4)$; $\text{flex}\left(16, \frac{8}{\sqrt{3}}\right)$; $\text{asintoto: } x = 4$



17)

$$y = \sqrt{x^2+1} + 2x$$

$$\text{Asintoti: } y = 3x, y = x$$

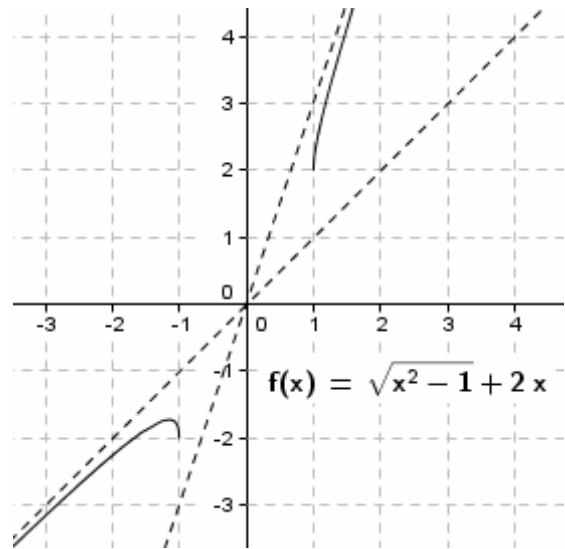


18)

$$y = \sqrt{x^2 - 1} + 2x$$

$$\text{Max}\left(-\frac{2}{\sqrt{3}}, -\sqrt{3}\right);$$

asintoti: $y = 3x$, $y = x$

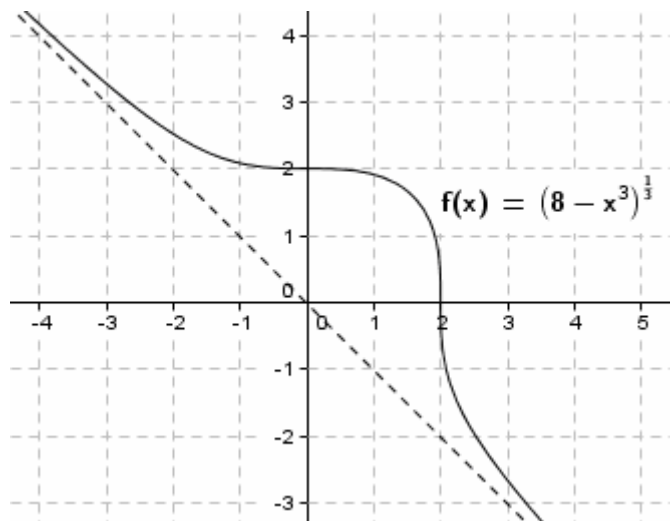


19)

$$y = \sqrt[3]{8 - x^3}$$

Flessi: $(0, 2)$ orizzontale, $(2, 0)$ verticale;

asintoto $y = -x$

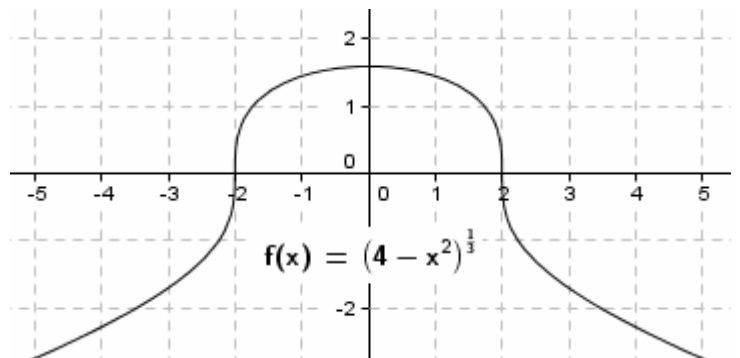


20)

$$y = \sqrt[3]{4 - x^2}$$

$$\text{Max}\left(0, \sqrt[3]{4}\right);$$

due flessi verticali in $(-2, 0)$ e $(2, 0)$



21)

$$y = \frac{x}{\sqrt[3]{x^2 - 1}}$$

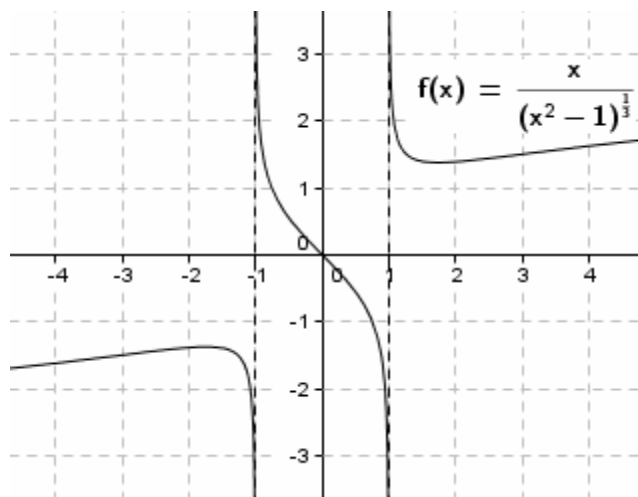
$$\text{Max}\left(-\sqrt{3}, -\frac{\sqrt{3}}{\sqrt[3]{2}}\right); \text{min}\left(\sqrt{3}, \frac{\sqrt{3}}{\sqrt[3]{2}}\right);$$

Flex₁(0, 0);

Flex₂ $\left(-3, -\frac{3}{2}\right)$;

Flex₃ $\left(3, \frac{3}{2}\right)$;

asintoti: $x = \pm 1$



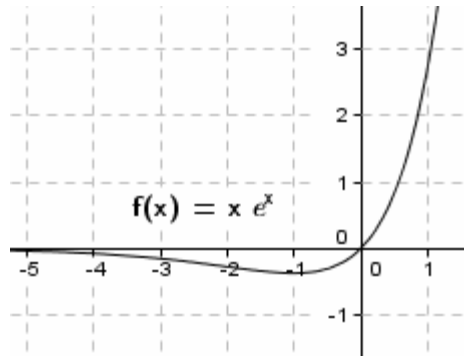
22)

$$y = xe^x$$

$$\min\left(-1 - \frac{1}{e}\right);$$

$$\text{Flex}\left(-2, -\frac{2}{e^2}\right);$$

$$\text{asintoto} : y = 0$$



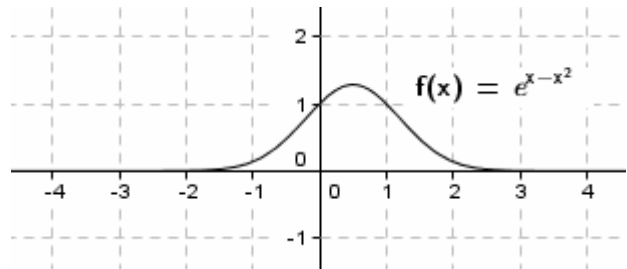
23)

$$y = e^{x-x^2}$$

$$\text{Max}\left(\frac{1}{2}, \sqrt[4]{e}\right);$$

$$\text{flessi}\left(\frac{1 \pm \sqrt{2}}{2}, \frac{1}{\sqrt[4]{e}}\right);$$

$$\text{asintoto} y = 0$$



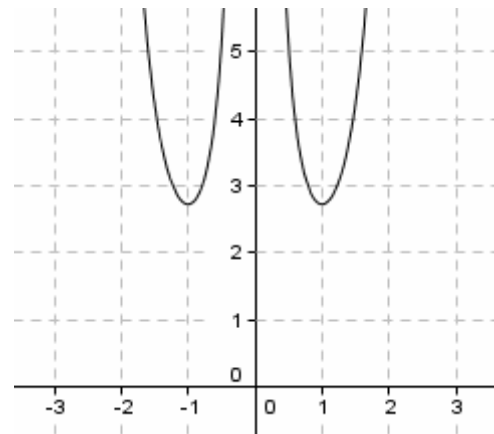
24)

$$y = \frac{e^{x^2}}{x^2}$$

$$\min_1(-1, e);$$

$$\min_2(1, e);$$

$$\text{asintoto} x = 0$$



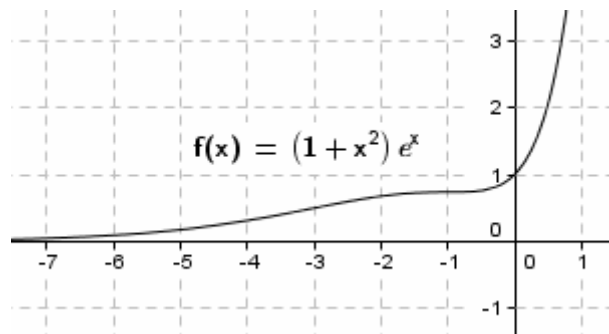
25)

$$y = (1+x^2)e^x$$

$$\text{Flex}_1\left(-3, \frac{10}{e^3}\right);$$

$$\text{Flex}_2\left(-1, \frac{2}{e}\right) \text{ (orizzontale)};$$

$$\text{asintoto} y = 0$$



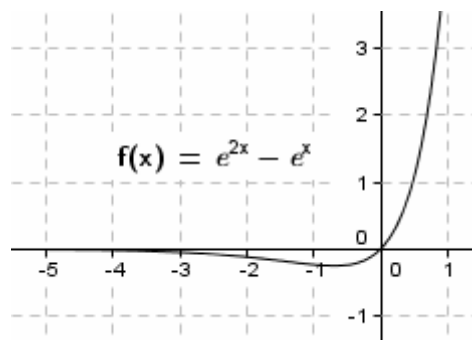
26)

$$y = e^{2x} - e^x$$

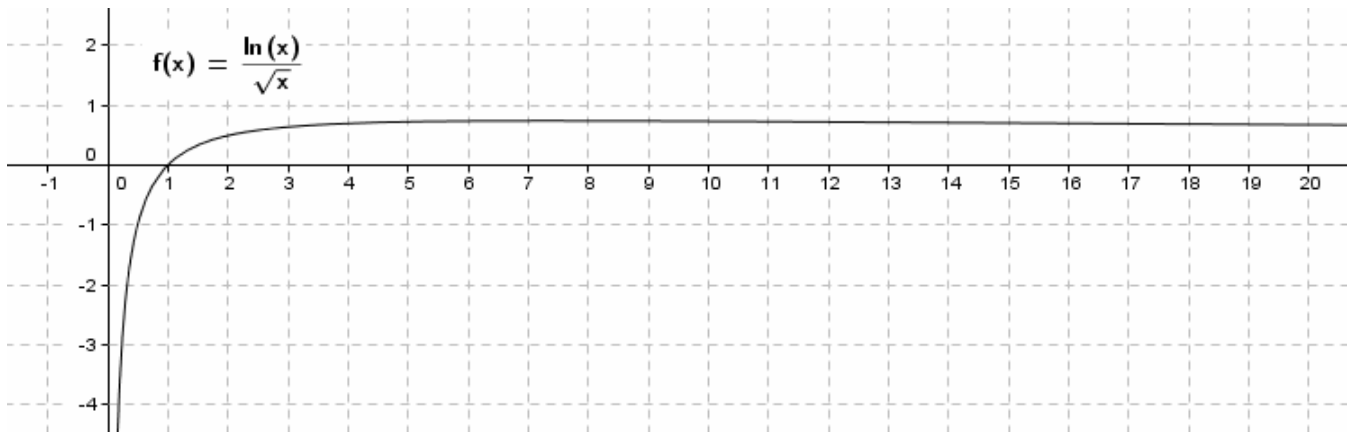
$$\min\left(-\ln 2, -\frac{1}{4}\right);$$

$$\text{Flex}\left(-2\ln 2, -\frac{3}{16}\right)$$

$$\text{asintoto} y = 0$$



27) $y = \frac{\ln x}{\sqrt{x}}$ $\text{Max}\left(e^2, \frac{2}{e}\right)$; $\text{Flex}\left(e^2 \sqrt[3]{e^2} \approx 14.39, \frac{8}{3e\sqrt[3]{e}} \approx 0.70\right)$; *asintoti* $x=0, y=0$



28)

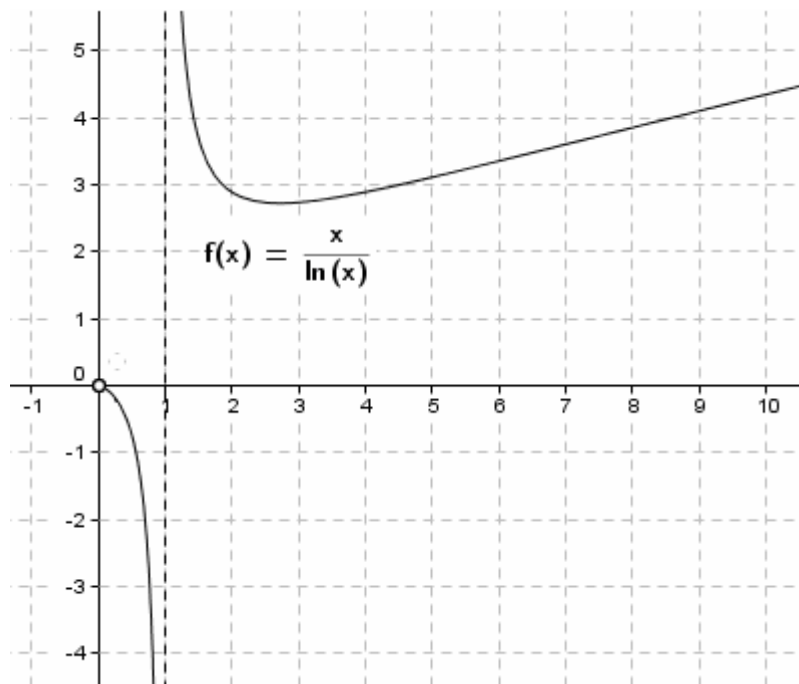
$$y = \frac{x}{\ln x}$$

$\min(e, e)$;

$$\text{Flex}\left(e^2, \frac{e^2}{2}\right)$$

asintoto : $x=1$;

$$\lim_{x \rightarrow 0^+} f(x) = 0$$



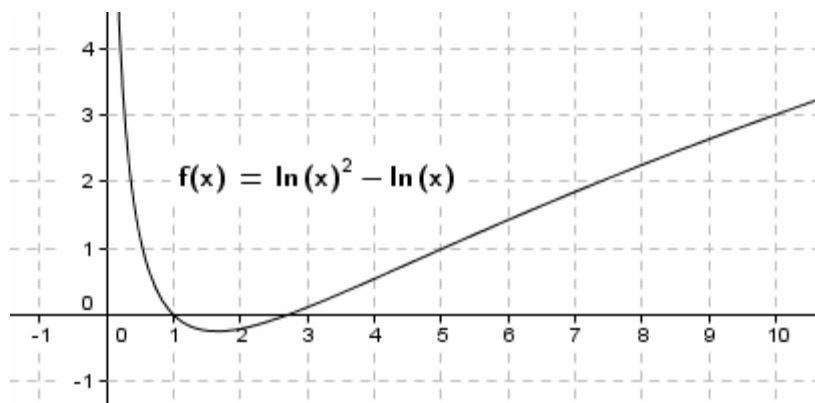
29)

$$y = \ln^2 x - \ln x$$

$$\min\left(\sqrt{e}, -\frac{1}{4}\right)$$

$$\text{Flex}\left(e\sqrt{e}, \frac{3}{4}\right)$$

asintoto : $x=0$



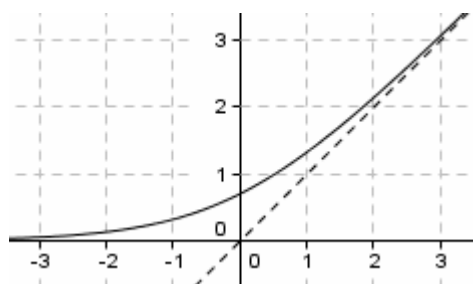
30)

$$y = \ln(1 + e^x)$$

Asintoti :

$y=0$ (a sinistra),

$y=x$ (a destra)



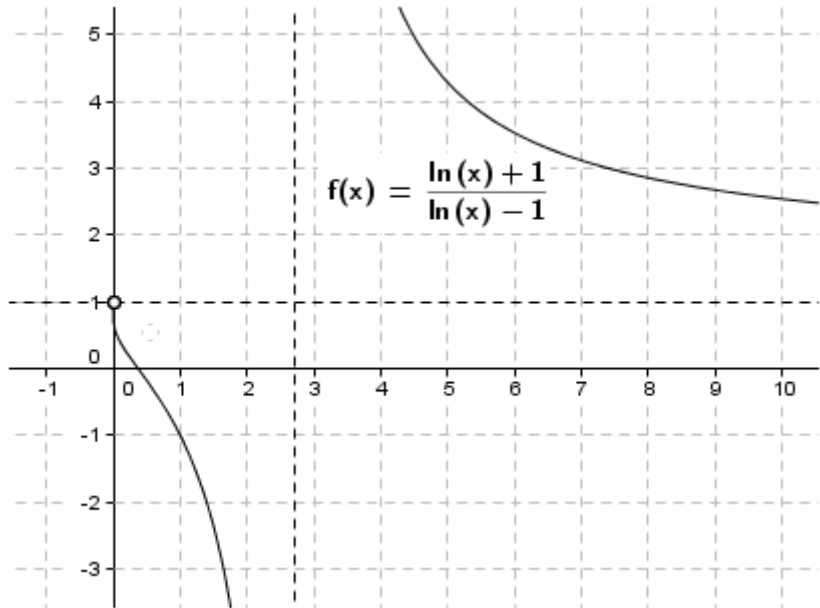
31)

$$y = \frac{\ln x + 1}{\ln x - 1}$$

$$\text{Flex} \left(\frac{1}{e}, 0 \right);$$

Asintoti : $x = e$, $y = 1$;

$$\lim_{x \rightarrow 0^+} f(x) = 1$$



32)

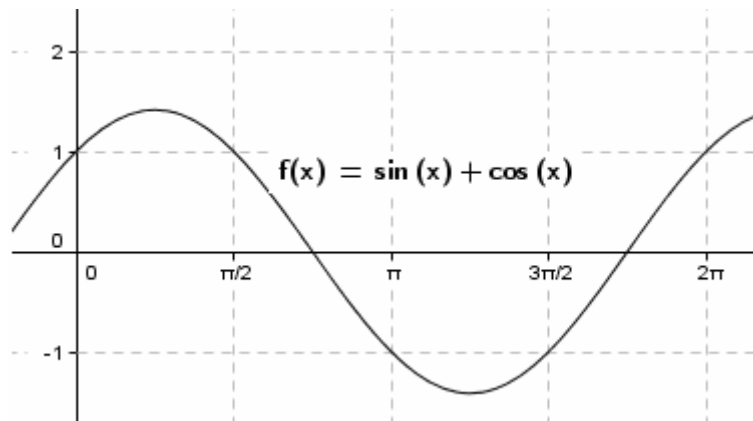
$$y = \sin x + \cos x \text{ su } [0, 2\pi]$$

$$\text{Max} \left(\frac{\pi}{4}, \sqrt{2} \right);$$

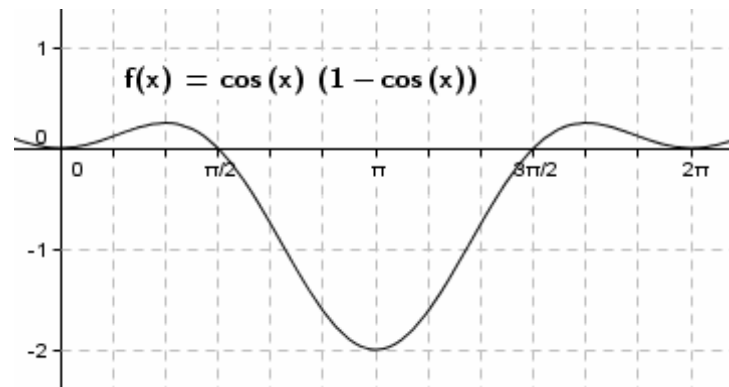
$$\text{min} \left(\frac{5}{4}\pi, -\sqrt{2} \right);$$

$$\text{Flex}_1 \left(\frac{3}{4}\pi, 0 \right);$$

$$\text{Flex}_2 \left(\frac{7}{4}\pi, 0 \right)$$



33)



$$y = \cos x(1 - \cos x) \text{ su } [0, 2\pi]$$

$$\text{Max}_1\left(\frac{\pi}{3}, \frac{1}{4}\right);$$

$$\text{Max}_2\left(\frac{5}{3}\pi, \frac{1}{4}\right);$$

$$\text{min}_1(0, 0);$$

$$\text{min}_2(\pi, -2);$$

$$\text{min}_3(2\pi, 0);$$

4 flessi, di ascisse

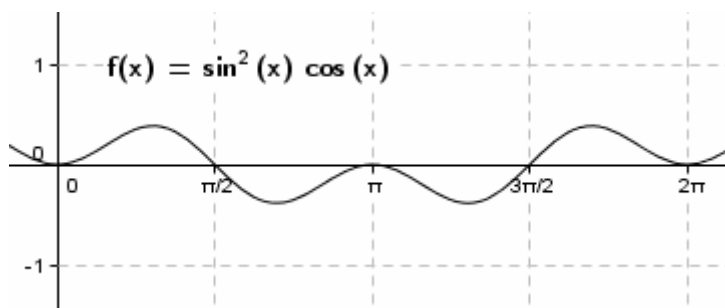
$$\arccos \frac{1 + \sqrt{33}}{8},$$

$$\arccos \frac{1 - \sqrt{33}}{8},$$

$$2\pi - \arccos \frac{1 - \sqrt{33}}{8},$$

$$2\pi - \arccos \frac{1 + \sqrt{33}}{8}$$

34)



$$y = \operatorname{sen}^2 x \cos x \quad \text{su } [0, 2\pi]$$

$$\min_1(0,0)$$

$$\operatorname{Max}_1\left(\operatorname{arc\,sen}\frac{\sqrt{6}}{3}, \frac{2}{3\sqrt{3}}\right);$$

$$\min_2\left(\pi - \operatorname{arc\,sen}\frac{\sqrt{6}}{3}, -\frac{2}{3\sqrt{3}}\right);$$

$$\operatorname{Max}_2(\pi,0);$$

$$\min_3\left(\pi + \operatorname{arc\,sen}\frac{\sqrt{6}}{3}, -\frac{2}{3\sqrt{3}}\right);$$

$$\operatorname{Max}_3\left(2\pi - \operatorname{arc\,sen}\frac{\sqrt{6}}{3}, \frac{2}{3\sqrt{3}}\right);$$

$$\min_4(2\pi,0)$$

$$\operatorname{Flex}_1\left(\operatorname{arc\,sen}\frac{\sqrt{2}}{3}, \frac{2\sqrt{7}}{27}\right);$$

$$\operatorname{Flex}_2\left(\frac{\pi}{2}, 0\right);$$

$$\operatorname{Flex}_3\left(\pi - \operatorname{arc\,sen}\frac{\sqrt{2}}{3}, -\frac{2\sqrt{7}}{27}\right);$$

$$\operatorname{Flex}_4\left(\pi + \operatorname{arc\,sen}\frac{\sqrt{2}}{3}, -\frac{2\sqrt{7}}{27}\right);$$

$$\operatorname{Flex}_5\left(\frac{3}{2}\pi, 0\right);$$

$$\operatorname{Flex}_6\left(2\pi - \operatorname{arc\,sen}\frac{\sqrt{2}}{3}, \frac{2\sqrt{7}}{27}\right)$$

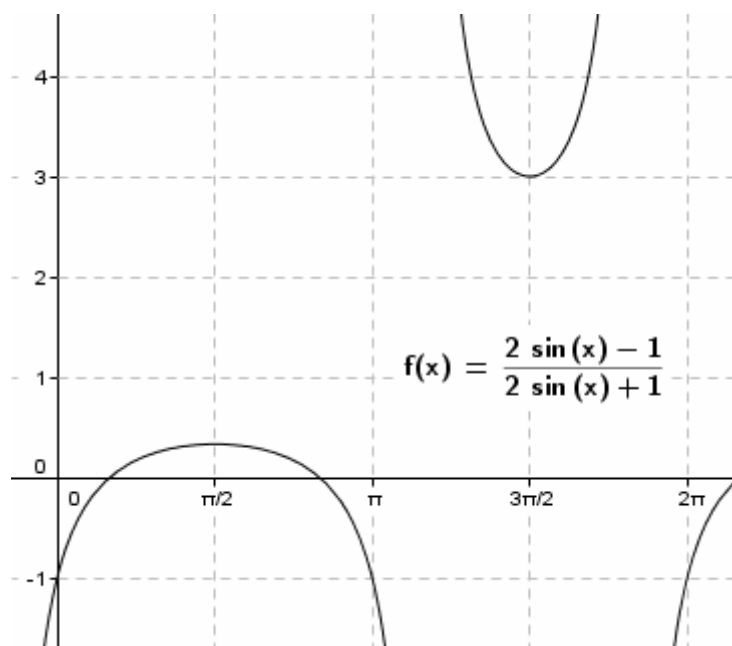
35)

$$y = \frac{2\operatorname{sen} x - 1}{2\operatorname{sen} x + 1} \quad \text{su } [0, 2\pi]$$

$$\operatorname{Max}\left(\frac{\pi}{2}, \frac{1}{3}\right);$$

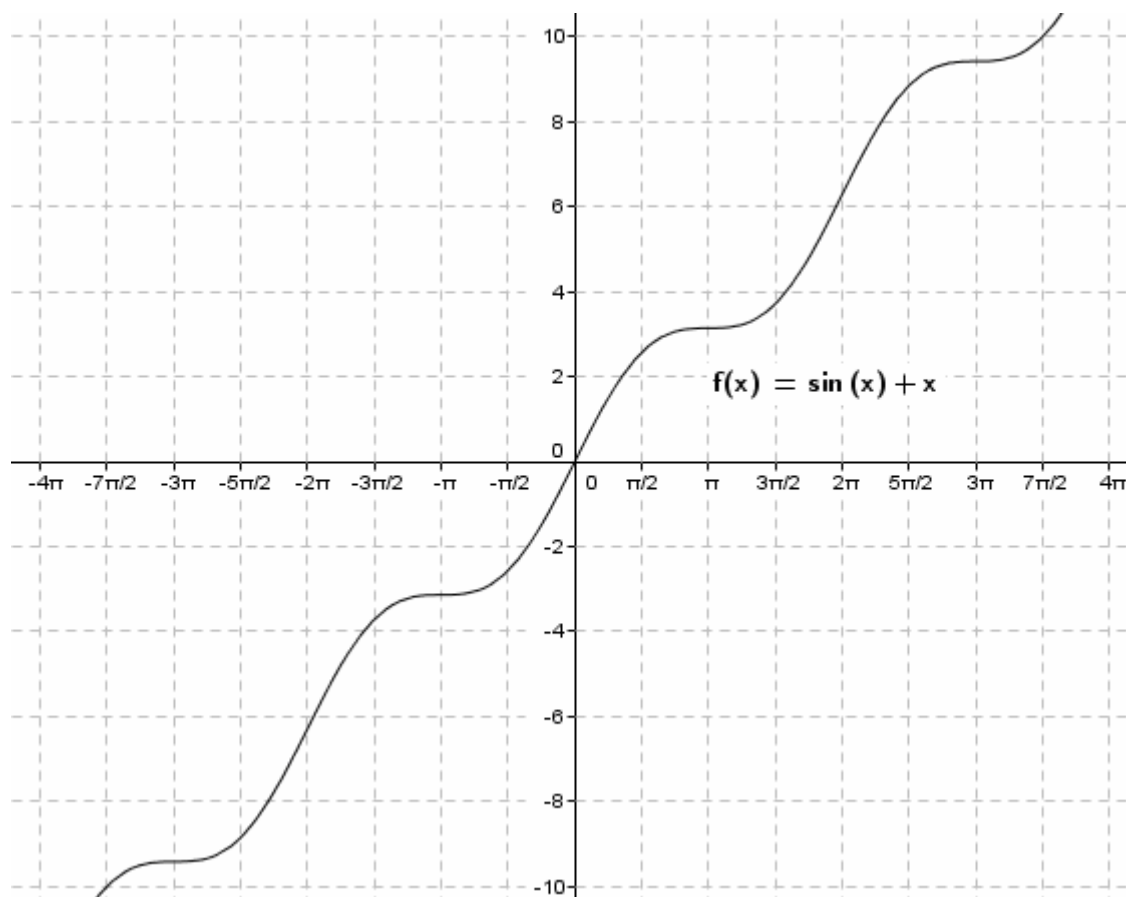
$$\min\left(\frac{3}{2}\pi, 3\right);$$

$$\text{asintoti} : x = \frac{7}{6}\pi, x = \frac{11}{6}\pi$$



36) $y = \sin x + x$

Questa funzione non è periodica; i suoi punti di flesso sono quelli di ascisse $x = k\pi$, con $k \in \mathbb{Z}$

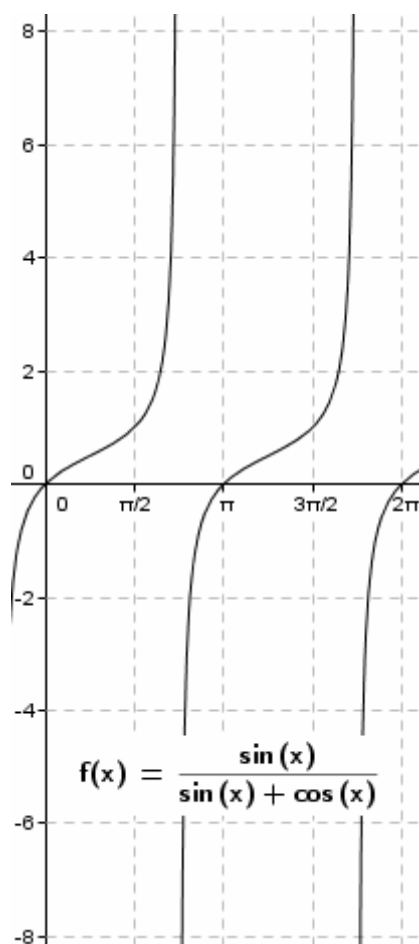


37)

$$y = \frac{\sin x}{\sin x + \cos x} \quad \text{su } [0, 2\pi]$$

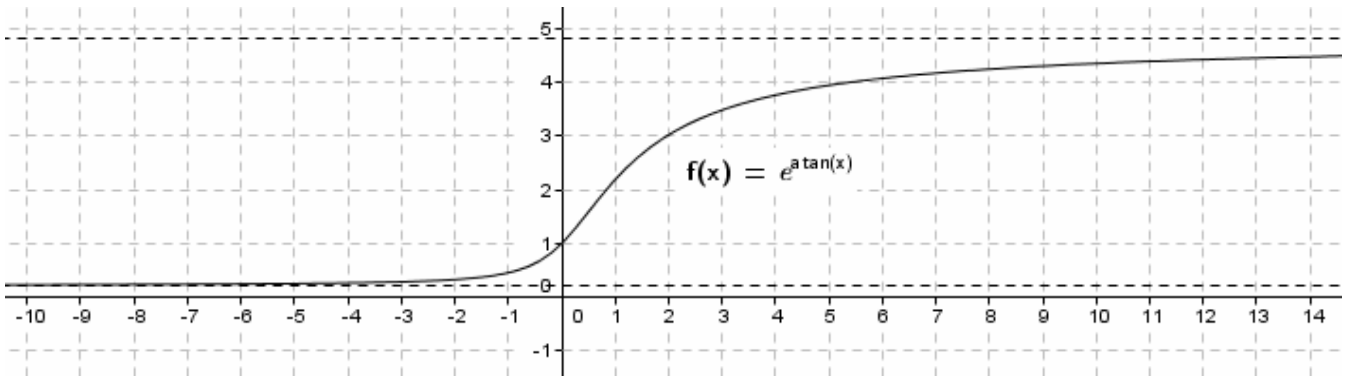
$$\text{Flessi: } \left(\frac{\pi}{4}, \frac{1}{2}\right), \left(\frac{5\pi}{4}, \frac{1}{2}\right);$$

$$\text{asintoti: } x = \frac{3}{4}\pi, x = \frac{7}{4}\pi$$



38)

$$y = e^{\arctg x} \text{ su } [0, 2\pi] \quad \text{Flex} \left(\frac{1}{2}, e^{\arctg 0.5} \approx 1.59 \right); \quad \text{asintoti: } y = e^{-\frac{\pi}{2}} \approx 0.21 \text{ (sinistro)}, y = e^{\frac{\pi}{2}} \approx 4.81 \text{ (destra)}$$



39)

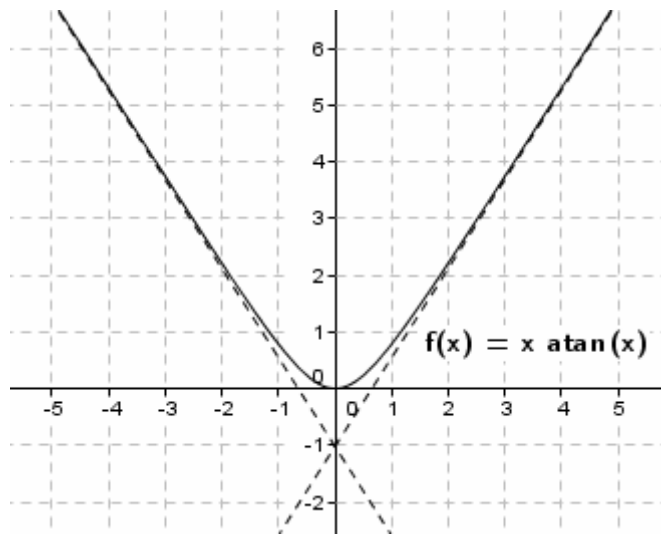
$$y = x \arctg x$$

$$\min(0,0);$$

asintoti :

$$y = -\frac{\pi}{2}x - 1 \text{ (sinistro)}$$

$$y = \frac{\pi}{2}x - 1 \text{ (destra)}$$



40)

$$y = x - 2 \arctg x$$

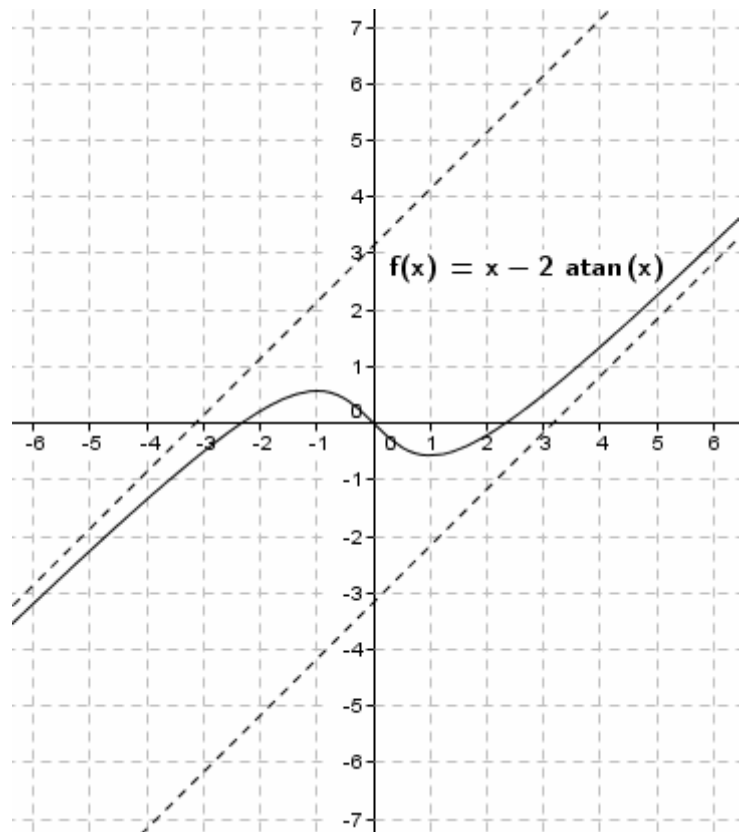
$$\text{Max} \left(-1, \frac{\pi}{2} - 1 \right);$$

$$\text{min} \left(1, 1 - \frac{\pi}{2} \right);$$

asintoti :

$$y = x + \pi \text{ (sinistro)}$$

$$y = x - \pi \text{ (destra)}$$



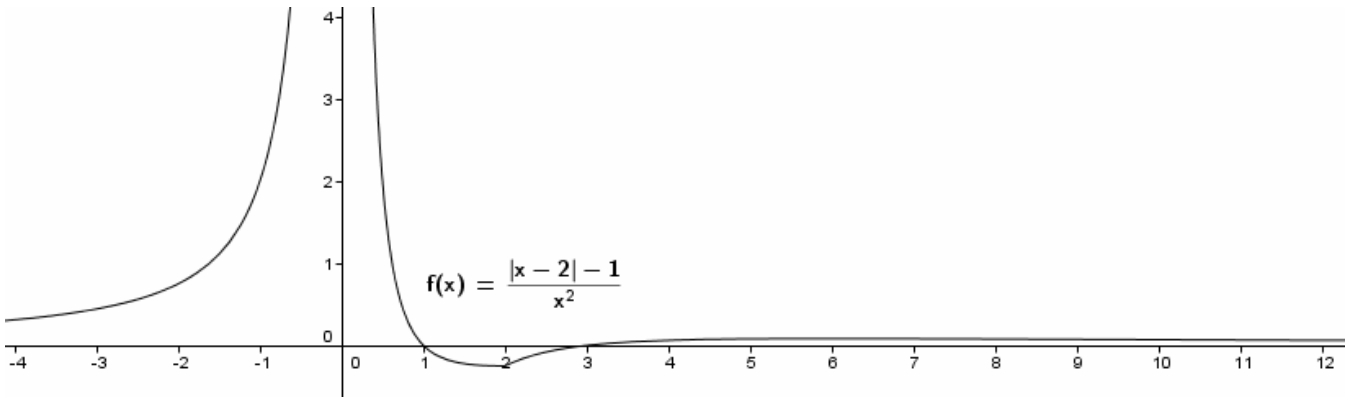
41)

$$y = \frac{|x-2|-1}{x^2}$$

$$\min\left(2, -\frac{1}{4}\right);$$

$$\text{Max}\left(6, \frac{1}{12}\right);$$

asintoti : $x = 0, y = 0$



42)

$$y = \frac{x|x+2|}{2(|x|-3)}$$

$$\text{Max}_1(-3-\sqrt{3}, -2-\sqrt{3});$$

$$\min_1(-2, 0);$$

$$\text{Max}_2(-3+\sqrt{3}, 2-\sqrt{3});$$

$$\min_2(3+\sqrt{15}, 4+\sqrt{15});$$

asintoti :

$$x = \pm 3;$$

$$x = \frac{1}{2}x + \frac{5}{2};$$

$$x = \frac{1}{2}x - \frac{1}{2}$$

