

1)

$$\begin{aligned} & \frac{1}{2} + \left(\frac{2x+1}{2x+4} - \frac{x^2+1}{x^2+4x+4} \right) \cdot \left(x+4 + \frac{4}{x} \right) = \\ & = \frac{1}{2} + \left[\frac{2x+1}{2(x+2)} - \frac{x^2+1}{(x+2)^2} \right] \cdot \frac{x^2+4x+4}{x} = \\ & = \frac{1}{2} + \frac{(x+2)(2x+1) - 2(x^2+1)}{2(x+2)^2} \cdot \frac{(x+2)^2}{x} = \\ & = \frac{1}{2} + \frac{\cancel{2x^2} + x + 4x + \cancel{2} - \cancel{2x^2} - \cancel{2}}{2(x+2)^2} \cdot \frac{\cancel{(x+2)^2}}{x} = \\ & = \frac{1}{2} + \frac{\cancel{5x} + \cancel{1}}{\cancel{2}} \cdot \frac{1}{\cancel{x}} = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3 \end{aligned}$$

2)

$$\begin{aligned} & \left(\frac{1}{a^2+3a+2} - \frac{1}{a^2+4a+3} \right) (a^2+5a+6) = \left[\frac{1}{(a+1)(a+2)} - \frac{1}{(a+1)(a+3)} \right] (a+2)(a+3) = \\ & = \frac{\cancel{a+3} - \cancel{a} - 2}{(a+1)\cancel{(a+2)}\cancel{(a+3)}} \cdot \cancel{(a+2)}\cancel{(a+3)} = \frac{1}{a+1} \end{aligned}$$

3)

$$\begin{aligned} & \left(\frac{a}{a^2-1} - \frac{a}{a^2+2a+1} \right) \cdot \frac{a^3+a^2-a-1}{a} = \\ & = \left[\frac{a}{(a+1)(a-1)} - \frac{a}{(a+1)^2} \right] \cdot \frac{a^2(a+1) - (a+1)}{a} = \\ & = \frac{a(a+1) - a(a-1)}{(a+1)^2(a-1)} \cdot \frac{(a+1)(a^2-1)}{a} = \\ & = \frac{\cancel{a^2} + a - \cancel{a^2} + a}{(a+1)^2(a-1)} \cdot \frac{(a+1)\cancel{(a+1)}\cancel{(a-1)}}{a} = \\ & = \frac{\cancel{2a}}{\cancel{a}} = 2 \end{aligned}$$

4)

$$\begin{aligned} & \left(\frac{x}{x+4} + \frac{x+4}{x} - 2 \right) \left(\frac{x+4}{4} - 1 \right) = \frac{x^2 + (x+4)^2 - 2x(x+4)}{x(x+4)} \cdot \frac{\cancel{x+4} - \cancel{4}}{4} = \\ & = \frac{\cancel{x^2} + \cancel{x^2} + 8x + 16 - \cancel{2x^2} - \cancel{8x}}{x(x+4)} \cdot \frac{4}{\cancel{x}(x+4)\cancel{4}} = \frac{4}{x+4} \end{aligned}$$

8)

$$\begin{aligned} & \left(\frac{1}{y^2-4y} - \frac{8}{y^3-4y^2} + \frac{16}{y^4-4y^3} \right) : \left(\frac{1}{y^2} - 2 \cdot \frac{2}{y^3} \right) = \\ & = \left[\frac{1}{y(y-4)} - \frac{8}{y^2(y-4)} + \frac{16}{y^3(y-4)} \right] : \left(\frac{1}{y^2} - \frac{4}{y^3} \right) = \frac{y^2-8y+16}{\cancel{y^3}(y-4)} \cdot \frac{\cancel{y^3}}{y-4} = \frac{\cancel{(y-4)^2}}{\cancel{(y-4)^2}} = 1 \end{aligned}$$

9)

$$\begin{aligned} & \left(\frac{n-2}{3n^2-12n+9} - \frac{1}{6n-18} \right) (6n-6) = \left[\frac{n-2}{3(n^2-4n+3)} - \frac{1}{6(n-3)} \right] \cdot 6(n-1) = \\ & = \left[\frac{n-2}{3(n-1)(n-3)} - \frac{1}{6(n-3)} \right] \cdot 6(n-1) = \frac{2n-4-n+1}{6(n-1)(n-3)} \cdot \cancel{6(n-1)} = \frac{n-3}{n-3} = 1 \end{aligned}$$

16)

$$\begin{aligned} & \left(\frac{1}{x^3-x^2-x+1} - \frac{1}{x^3+x^2-x-1} \right) (x^2-1)^2 = \\ & = \left[\frac{1}{x^2(x-1)-(x-1)} - \frac{1}{x^2(x+1)-(x+1)} \right] (x+1)^2(x-1)^2 = \\ & = \left[\frac{1}{(x-1)(x^2-1)} - \frac{1}{(x+1)(x^2-1)} \right] (x+1)^2(x-1)^2 = \\ & = \left[\frac{1}{(x-1)(x+1)(x-1)} - \frac{1}{(x+1)(x+1)(x-1)} \right] (x+1)^2(x-1)^2 = \\ & = \left[\frac{1}{(x-1)^2(x+1)} - \frac{1}{(x+1)^2(x-1)} \right] (x+1)^2(x-1)^2 = \frac{\cancel{x+1}\cancel{x+1}}{(x-1)^2(x+1)^2} \cdot \cancel{(x+1)^2(x-1)^2} = 2 \end{aligned}$$

18)

$$\begin{aligned} & \left(a \cdot \frac{5a+4}{4a^3-4a^2-a+1} - \frac{2a+1}{2a^2-3a+1} \right) \cdot (4a^2-1) = \left[\frac{a(5a+4)}{4a^2(a-1)-(a-1)} - \frac{2a+1}{2a^2-2a-a+1} \right] \cdot (4a^2-1) = \\ & = \left[\frac{a(5a+4)}{(a-1)(4a^2-1)} - \frac{2a+1}{2a(a-1)-(a-1)} \right] \cdot (4a^2-1) = \left[\frac{a(5a+4)}{(a-1)(2a+1)(2a-1)} - \frac{2a+1}{(a-1)(2a-1)} \right] \cdot (4a^2-1) = \\ & = \frac{a(5a+4)-(2a+1)^2}{(a-1)(2a+1)(2a-1)} \cdot \cancel{(2a+1)(2a-1)} = \frac{5a^2-4a-4a^2-4a-1}{a-1} = \frac{a^2-1}{a-1} = \frac{(a+1)\cancel{(a-1)}}{\cancel{a-1}} = a+1 \end{aligned}$$

22)

$$\begin{aligned} & \left(\frac{1}{4a-8} - \frac{1}{6a-12} \right) \cdot (12a^2-12a-24) - a = \\ & = \left[\frac{1}{4(a-2)} - \frac{1}{6(a-2)} \right] \cdot 12(a^2-a-2) - a = \\ & = \frac{3-2}{12(a-2)} \cdot \cancel{12(a-2)} - a = \cancel{a+1}\cancel{a} = 1 \end{aligned}$$

24)

$$\frac{\frac{a-1}{a+1} - \frac{a+1}{a-1}}{\frac{1}{a+1} - 1} = \frac{\frac{(a-1)^2 - (a+1)^2}{(a+1)(a-1)}}{\frac{\cancel{1-a}\cancel{1}}{a+1}} = \frac{\cancel{a^2} - 2a\cancel{1} - \cancel{a^2} - 2a\cancel{1}}{(\cancel{a+1})(a-1)} \cdot \frac{\cancel{a+1}}{-a} = \frac{-4\cancel{a}}{-\cancel{a}(a-1)} = \frac{4}{a-1}$$

26)

$$\begin{aligned}
 & \frac{b^3}{b^6 + b^4 + b^2 + 1} - \frac{1}{b^3 + b^2 + b + 1} \cdot (b^8 - 1) + 1 = \\
 & = \frac{b^3}{b^4(b^2 + 1) + (b^2 + 1)} - \frac{1}{b^2(b + 1) + (b + 1)} \cdot (b^4 + 1)(b^4 - 1) + 1 = \\
 & = \frac{b^3}{(b^2 + 1)(b^4 + 1)} - \frac{1}{(b + 1)(b^2 + 1)} \cdot (b^4 + 1)(b^2 + 1)(b^2 - 1) + 1 = \\
 & = \frac{b^3(b + 1) - (b^4 + 1)}{(b + 1)(b^2 + 1)(b^4 + 1)} \cdot (b^4 + 1)(b^2 + 1)(b + 1)(b - 1) + 1 = \\
 & = \frac{\cancel{b^4} + b^3 - \cancel{b^4} - 1}{(b + 1)(b^2 + 1)(b^4 + 1)} \cdot (b^4 + 1)(b^2 + 1)(b + 1)(b - 1) + 1 = \\
 & = \frac{b^3 - 1}{(b + 1)(b^2 + 1)(b^4 + 1)} \cdot \frac{1}{b - 1} \cdot (\cancel{b^4 + 1})(\cancel{b^2 + 1})(\cancel{b + 1})(\cancel{b - 1}) + 1 = b^3 \cancel{1} \cancel{1} = b^3
 \end{aligned}$$

38)

$$\begin{aligned}
 & \frac{1}{\left(\frac{x}{x^2 - 3xy + 2y^2} + \frac{1}{2y - x}\right)\left(\frac{x}{y} - 2\right)} + y = \frac{1}{\left[\frac{x}{(x - y)(x - 2y)} - \frac{1}{x - 2y}\right] \cdot \frac{x - 2y}{y}} + y = \\
 & = \frac{1}{\cancel{x} \cancel{+ y} \cdot \frac{\cancel{x - 2y}}{y}} + y = \frac{1}{\cancel{y} \cdot \frac{1}{x - y} \cdot \cancel{y}} + y = x \cancel{y} \cancel{+ y} = x
 \end{aligned}$$

42)

$$\begin{aligned}
 & 3\left(\frac{x}{3x - 6} - \frac{x}{x^2 - x - 2}\right) \cdot (x^2 + x^1 + x^0 + x^{-1}) - x^2 = 3\left[\frac{x}{3(x - 2)} - \frac{x}{(x + 1)(x - 2)}\right] \cdot \left(x^2 + x + 1 + \frac{1}{x}\right) - x^2 = \\
 & = \cancel{3} \cdot \frac{x^2 + x - 3x}{\cancel{3}(x + 1)(x - 2)} \cdot \frac{x^3 + x^2 + x + 1}{x} - x^2 = \frac{x^2 - 2x}{(x + 1)(x - 2)} \cdot \frac{x^2(x + 1) + (x + 1)}{x} - x^2 = \\
 & = \frac{\cancel{x}(x - 2)}{(x + 1)(x - 2)} \cdot \frac{(x + 1)(x^2 + 1)}{\cancel{x}} - x^2 = \cancel{x}^2 + 1 - \cancel{x}^2 = 1
 \end{aligned}$$

43)

$$\begin{aligned}
 & \left[\frac{1}{4}\left(\frac{1}{x + 4} - \frac{1}{x - 4}\right)\right]^2 \left(\frac{x^2 + 8x + 16}{2}\right)^2 = \left[\frac{1}{4}\frac{\cancel{x} - 4 - \cancel{x} - 4}{(x + 4)(x - 4)}\right]^2 \left[\frac{(x + 4)^2}{2}\right]^2 = \\
 & = \left[\frac{1}{\cancel{4}}\frac{-\cancel{8}^2}{(x + 4)(x - 4)}\right]^2 \frac{(x + 4)^4}{4} = \frac{\cancel{4}}{(x + 4)^2(x - 4)^2} \frac{(x + 4)^{\cancel{4}^2}}{\cancel{4}} = \frac{(x + 4)^2}{(x - 4)^2}
 \end{aligned}$$

44)

$$\begin{aligned} & \left(\frac{1}{a^2 - 2a + 1} - \frac{1}{a^2 + 2a + 1} \right) \cdot \frac{a^4 - 2a^2 + 1}{4} = \left[\frac{1}{(a-1)^2} - \frac{1}{(a+1)^2} \right] \cdot \frac{(a^2 - 1)^2}{4} = \\ & = \frac{(a+1)^2 - (a-1)^2}{(a-1)^2 (a+1)^2} \cdot \frac{[(a+1)(a-1)]^2}{4} = \\ & = \frac{\cancel{a^2} + 2a\cancel{1} - \cancel{a^2} + 2a\cancel{1}}{(a-1)^2 (a+1)^2} \cdot \frac{(a+1)^2 (a-1)^2}{4} = \frac{\cancel{4}a}{\cancel{4}} = a \end{aligned}$$

50)

$$\begin{aligned} & \left[6 \left(\frac{\frac{1}{2}a}{a-b} + \frac{\frac{1}{3}a}{b-a} \right) + \frac{ab^3}{a^4 - ab^3} \right] \cdot \left(a + b - \frac{ab}{a+b} \right) = \\ & = \left[6 \left(\frac{1}{2}a \cdot \frac{1}{a-b} + \frac{1}{3}a \cdot \frac{1}{b-a} \right) + \frac{\cancel{ab^3}}{\cancel{a^3 - b^3}} \right] \cdot \frac{(a+b)^2 - ab}{a+b} = \\ & = \left\{ 6 \left[\frac{a}{2(a-b)} + \frac{a}{3(b-a)} \right] + \frac{b^3}{a^3 - b^3} \right\} \cdot \frac{a^2 + 2ab + b^2 - ab}{a+b} = \\ & = \left\{ 6 \left[\frac{a}{2(a-b)} - \frac{a}{3(a-b)} \right] + \frac{b^3}{a^3 - b^3} \right\} \cdot \frac{a^2 + ab + b^2}{a+b} = \left\{ \cancel{6} \cdot \frac{3a - 2a}{\cancel{6}(a-b)} + \frac{b^3}{a^3 - b^3} \right\} \cdot \frac{a^2 + ab + b^2}{a+b} = \\ & = \left\{ \frac{a}{a-b} + \frac{b^3}{(a-b)(a^2 + ab + b^2)} \right\} \cdot \frac{a^2 + ab + b^2}{a+b} = \frac{a^3 + a^2b + ab^2 + b^3}{(a-b)(a^2 + ab + b^2)} \cdot \frac{\cancel{a^2 + ab + b^2}}{a+b} = \\ & = \frac{a^2(a+b) + b^2(a+b)}{(a-b)(a+b)} = \frac{\cancel{(a+b)}(a^2 + b^2)}{(a-b)\cancel{(a+b)}} = \frac{a^2 + b^2}{a-b} \end{aligned}$$

52)

$$\begin{aligned} & \frac{\frac{y+1}{y-1} + \frac{2}{(1-y)^2} + 1}{1-y+y^2} = \frac{\frac{y+1}{y-1} + \frac{2}{(y-1)^2} + 1}{1-y+y^2} = \frac{\cancel{y^2} + 2 + y^2 - 2y\cancel{1}}{(y-1)^2} = \\ & = \frac{2y^2 - 2y + 2}{(y-1)^2} \cdot \frac{1}{1-y+y^2} = \frac{2(\cancel{y^2 - y + 1})}{(y-1)^2} \cdot \frac{1}{\cancel{(1-y+y^2)}} = \frac{2}{(y-1)^2} \end{aligned}$$

55)

$$\begin{aligned} & 1 + \left(\frac{1}{n-1} - \frac{n}{n^2 - 1} \right) (n^4 - 2n^2 + 1) = 1 + \left[\frac{1}{n-1} - \frac{n}{(n+1)(n-1)} \right] (n^2 - 1)^2 = \\ & = 1 + \frac{\cancel{n} + 1 - \cancel{n}}{(n+1)\cancel{(n-1)}} \cdot (n+1)^2 (n-1)^2 = \cancel{1} + n^2 \cancel{1} = n^2 \end{aligned}$$

56)

$$\begin{aligned}
 & 1 + \left[\frac{1}{(n-1)(n^2+n+1)} - \frac{n}{n^4-1} \right] \left[n^3 + (n^2+1)^2 + n \right] = \\
 & = 1 + \left[\frac{1}{(n-1)(n^2+n+1)} - \frac{n}{(n^2+1)(n+1)(n-1)} \right] \cdot \left[n^3 + n + (n^2+1)^2 \right] = \\
 & = 1 + \frac{(n+1)(n^2+1) - n(n^2+n+1)}{(n+1)(n-1)(n^2+n+1)(n^2+1)} \cdot \left[n(n^2+1) + (n^2+1)^2 \right] = \\
 & = 1 + \frac{\cancel{n^3} + \cancel{n} + \cancel{n^2} + 1 - \cancel{n^3} - \cancel{n^2} - \cancel{n}}{(n+1)(n-1)\cancel{(n^2+n+1)}\cancel{(n^2+1)}} \cdot \cancel{(n^2+1)}\cancel{(n^2+n+1)} = \\
 & = 1 + \frac{1}{(n+1)(n-1)} = \frac{\cancel{n^2} \cancel{1}}{(n+1)(n-1)} = \frac{n^2}{(n+1)(n-1)}
 \end{aligned}$$

57)

$$\begin{aligned}
 & \left(\frac{1}{a^2-2ab-3b^2} + \frac{1}{2b^2+ab-a^2} \right) \cdot (6ab^2-5a^2b+a^3) = \\
 & = \left[\frac{1}{(a-3b)(a+b)} - \frac{1}{a^2-ab-2b^2} \right] \cdot (a^3-5a^2b+6ab^2) = \\
 & = \left[\frac{1}{(a-3b)(a+b)} - \frac{1}{(a-2b)(a+b)} \right] \cdot a(a^2-5ab+6b^2) = \\
 & = \frac{\cancel{a} - 2b - \cancel{a} + 3b}{(a-3b)\cancel{(a-2b)}(a+b)} \cdot a\cancel{(a-2b)}\cancel{(a-3b)} = \frac{b}{a+b} \cdot a = \frac{ab}{a+b}
 \end{aligned}$$

65)

$$\begin{aligned}
 & \left(\frac{x+2}{x^3-8} - \frac{1}{x^2+2x+4} \right) \left(\frac{x}{4} - \frac{1}{2} \right) = \left[\frac{x+2}{(x-2)(x^2+2x+4)} - \frac{1}{x^2+2x+4} \right] \cdot \frac{x-2}{4} = \\
 & = \frac{\cancel{x+2} - \cancel{x} + 2}{\cancel{(x-2)}(x^2+2x+4)} \cdot \frac{\cancel{x-2}}{4} = \frac{1}{x^2+2x+4}
 \end{aligned}$$

72)

$$\frac{a^4+2a^3-2a-1}{a^3-3a-2} - 1 \stackrel{\text{RUFFINI}}{=} \frac{(a+1)^2(a-1)}{(a-2)(a+1)^2} - 1 = \frac{a^2-1-a+2}{a-2} = \frac{a^2-a+1}{a-2}$$

73)

$$\frac{a^4+a^3+4a}{a^4+4a^3+4a^2} = \frac{\cancel{a}(a^3+a^2+4)}{a^2(a^2+4a+4)} \stackrel{\text{RUFFINI}}{=} \frac{\cancel{(a+2)}(a^2-a+2)}{a(a+2)^2} = \frac{a^2-a+2}{a(a+2)}$$

74)

$$\begin{aligned}
& \left(\frac{b}{a^2 - b^2 - 2b - 1} + \frac{1}{a^2 + ab + a} \right) \left(\frac{a^2}{b+1} - a \right) = \\
& = \left[\frac{b}{a^2 - (b+1)^2} + \frac{1}{a(a+b+1)} \right] \cdot \frac{a^2 - a(b+1)}{b+1} = \\
& = \left[\frac{b}{(a+b+1)(a-b-1)} + \frac{1}{a(a+b+1)} \right] \cdot \frac{a^2 - ab - a}{b+1} = \\
& = \frac{ab + a - b - 1}{\cancel{a(a+b+1)(a-b-1)}} \cdot \frac{\cancel{a(a-b-1)}}{b+1} = \\
& = \frac{a(b+1) - (b+1)}{a+b+1} \cdot \frac{1}{b+1} = \\
& = \frac{\cancel{(b+1)}(a-1)}{a+b+1} \cdot \frac{1}{\cancel{b+1}} = \frac{a-1}{a+b+1}
\end{aligned}$$

76)

$$\begin{aligned}
& \left(x \cdot \frac{x^3 - 5x^2 + 11x - 10}{2x^2 - 3x - 2} - 1 \right) \cdot \left(-2 - \frac{3}{x-1} \right)^2 - 2x^2 = \\
& = \left(\frac{x^4 - 5x^3 + 11x^2 - 10x - 2x^2 + 3x + 2}{2x^2 - 3x - 2} \right) \cdot \left(\frac{-2x + 2 - 3}{x-1} \right)^2 - 2x^2 = \\
& = \left[\frac{x^4 - 5x^3 + 9x^2 - 7x + 2}{2x^2 - 3x - 2} \right] \cdot \left(\frac{-2x-1}{x-1} \right)^2 - 2x^2 \stackrel{\text{RUFFINI}}{=} \frac{(x-1)^3 \cancel{(x-2)}}{\cancel{(x-2)}(2x+1)} \cdot \frac{(-2x-1)^2}{(x-1)^2} - 2x^2 = \\
& = \frac{(x-1)^{\cancel{3}} (2x+1)^{\cancel{2}}}{\cancel{2x+1} \cancel{(x-1)^2}} - 2x^2 = \cancel{2x^2} + x - 2x - 1 - \cancel{2x^2} = -x - 1 = -(x+1)
\end{aligned}$$

80)

$$\begin{aligned}
& \left[\frac{b^4}{a(a-b)^2} - a \cdot \left(\frac{a}{b-a} \right)^2 \right] : \left(a^3 - a^{-1}b^4 \right) + \frac{1}{(a-b)^2} = \\
& = \left[\frac{b^4}{a(a-b)^2} - a \cdot \frac{a^2}{(b-a)^2} \right] : \left(a^3 - \frac{1}{a} \cdot b^4 \right) + \frac{1}{(a-b)^2} = \\
& = \left[\frac{b^4}{a(a-b)^2} - \frac{a^3}{(a-b)^2} \right] : \left(a^3 - \frac{b^4}{a} \right) + \frac{1}{(a-b)^2} = \\
& = \frac{b^4 - a^4}{a(a-b)^2} : \frac{a^4 - b^4}{a} + \frac{1}{(a-b)^2} = \\
& = \frac{\cancel{b^4 - a^4}^{-1}}{\cancel{a(a-b)^2} \cdot \cancel{a^4 - b^4}} + \frac{1}{(a-b)^2} = \\
& = \frac{\cancel{1}}{\cancel{(a-b)^2}} + \frac{\cancel{1}}{\cancel{(a-b)^2}} = 0
\end{aligned}$$