ESPRESSIONI CON PRODOTTI NOTEVOLI - CORREZIONI

9) $\left[(a-10b)^2-3(2a-5b)^2-5\left(5b^2-2a^2\right)\right]^2+10\cdot(2a)^3\cdot b=$

$=\left[a^2-20ab+100b^2-3\left(4a^2-20ab+25b^2\right)-25b^2+10a^2\right]^2+10\cdot 8a^3\cdot b=$

$=\left(\underline{a^2}\ \underline{\underline{-20ab}}\ \cancel{+100b^2}\ \underline{-12a^2}\ \underline{\underline{+60ab}}\ \cancel{-75b^2}\ \cancel{-25b^2}\ \underline{+10a^2}\right)^2+80a^3b=$

$=\left(-a^2+40ab\right)^2+80a^3b=a^4\cancel{-80a^3b}+1600a^2b^2\cancel{+80a^3b}=a^4+1600a^2b^2$

10) $\frac{1}{2}\left[(-2+x)^2+(-2-x)^2\right]\left(x^2-3\right)-\left(x^2+2\right)\left(x^2-6\right)=$

$=\frac{1}{2}\left[4\cancel{-4x}+x^2+4\cancel{+4x}+x^2\right]\left(x^2-3\right)-\left(x^4-6x^2+2x^2-12\right)=$

$=\frac{1}{2}\left(8+2x^2\right)\left(x^2-3\right)-\left(x^4-4x^2-12\right)=$

$\underset{NOTA}{=}\left(4+x^2\right)\left(x^2-3\right)-x^4+4x^2+12=$

$=\underline{4x^2}\ \cancel{-12}\ \cancel{+x^4}\ \underline{-3x^2}\ \cancel{-x^4}\ \underline{+4x^2}\ \cancel{+12}=5x^2$

NOTA
In generale, quando sia ha un monomio moltiplicato per due polinomi, è più conveniente lasciare il monomio indicato e moltiplicare prima fra loro i polinomi; ma questa espressione fa eccezione perché è decisamente più comodo, qui, moltiplicare innanzitutto il monomio per il primo polinomio.

11) $\left(-y^2-1\right)^2\left[2(-4+y)^2-(y-8)^2+31\right]^2=$

$=\left(y^4+2y^2+1\right)\left[2\left(16-8y+y^2\right)-\left(y^2-16y+64\right)+31\right]^2=$

$=\left(y^4+2y^2+1\right)\left[\underline{32}\ \cancel{-16y}\ \underline{\underline{+2y^2}}\ \underline{\underline{-y^2}}\ \cancel{+16y}\ \underline{-64}\ \underline{+31}\right]^2=$

$=\left(y^4+2y^2+1\right)\left(-1+y^2\right)^2=$

$=\left(y^4+2y^2+1\right)\left(1-2y^2+y^4\right)\underset{NOTA}{=}$

$=\underline{y^4}\ \cancel{-2y^6}+y^8\ \cancel{+2y^2}\ \underline{-4y^4}\ \cancel{+2y^6}+1\ \cancel{-2y^2}\ \underline{+y^4}=$

$=-2y^4+y^8+1\underset{\substack{\text{ordiniamo,}\\ \text{solo per}\\ \text{motivi di}\\ \text{eleganza}}}{=}y^8-2y^4+1$

NOTA
Meglio:

$\left(y^4+2y^2+1\right)\left(1-2y^2+y^4\right)=$

$=\left(y^4+1+2y^2\right)\left(y^4+1-2y^2\right)=$

$=\left(y^4+1\right)^2-\left(2y^2\right)^2=$

$=y^8+2y^4+1-4y^4=y^8-2y^4+1$

E meglio ancora (dal passaggio precedente):

$\left(y^4+2y^2+1\right)\left(-1+y^2\right)^2=$

$=\left(y^2+1\right)^2\left(y^2-1\right)^2=$

$=\left[\left(y^2+1\right)\left(y^2-1\right)\right]^2=$

$=\left(y^4-1\right)^2=y^8-2y^4+1$

12) $\left(3a-\frac{1}{2}\right)^2+\left(\frac{1}{2}a+1\right)\left(1+\frac{1}{2}a\right)-(a-1)^2-\frac{1}{4}\left(a^2+1\right)=$

$=9a^2-3a\cancel{+\frac{1}{4}}+\left(\frac{1}{2}a+1\right)^2-\left(a^2-2a+1\right)-\frac{1}{4}a^2\cancel{-\frac{1}{4}}=$

$=\underline{9a^2}\ \underline{\underline{\cancel{-3a}}}\ \cancel{+\frac{1}{4}a^2}\ \underline{\underline{\cancel{+a}}}\ \cancel{+1}\ \underline{-a^2}\ \underline{\underline{\cancel{+2a}}}\ \cancel{-1}\ \cancel{-\frac{1}{4}a^2}=8a^2$

21) $$(5a-4b)(5a+4b)-(3a+5b)(3a-5b)-(4a-3b)^2\underset{NOTA}{=}$$
$$=25a^2-16b^2-\left(9a^2-25b^2\right)-\left(16a^2-24ab+9b^2\right)=$$
$$=\underline{\cancel{25a^2}}\ \underline{\underline{\cancel{-16b^2}}}\ \underline{\cancel{-9a^2}}\ \underline{\underline{\cancel{+25b^2}}}\ \underline{\cancel{-16a^2}}+24ab\ \underline{\underline{\cancel{-9b^2}}}=24ab$$

NOTA
In realtà, questo primo passaggio
si sarebbe potuto anche saltare
facendo a mente
il cambiamento di segno
dovuto al “–“ davanti:
appena trovo un termine,
lo scrivo subito cambiato di segno.

26) $$\left[\left(\frac{1}{3}w+1\right)^2-\left(\frac{1}{3}w-1\right)^2+\underbrace{\left(-\frac{1}{3}w+1\right)\left(-\frac{1}{3}w-1\right)}_{NOTA}-\frac{4}{3}w\right]\left(\frac{1}{9}w^2+1\right)+1=$$
$$=\left[\frac{1}{9}w^2+\frac{2}{3}w+1-\left(\frac{1}{9}w^2-\frac{2}{3}w+1\right)+\frac{1}{9}w^2+\cancel{\frac{1}{3}w}\cancel{-\frac{1}{3}w}-1-\frac{4}{3}w\right]\left(\frac{1}{9}w^2+1\right)+1=$$
$$=\left[\cancel{\frac{1}{9}w^2}\ \underline{\cancel{+\frac{2}{3}w}}\ \cancel{+1}-\cancel{\frac{1}{9}w^2}\ \underline{\cancel{+\frac{2}{3}w}}\ \cancel{-1}+\frac{1}{9}w^2-1\ \underline{\cancel{-\frac{4}{3}w}}\right]\left(\frac{1}{9}w^2+1\right)+1=$$
$$=\left(\frac{1}{9}w^2-1\right)\left(\frac{1}{9}w^2+1\right)+1=\frac{1}{81}w^4\ \cancel{-1}\ \cancel{+1}=\frac{1}{81}w^4$$

NOTA: $\left(-\frac{1}{3}w+1\right)\left(-\frac{1}{3}w-1\right)$ può anche essere svolto come un prodotto notevole:

$$\left(\boxed{-\frac{1}{3}w}+1\right)\left(\boxed{-\frac{1}{3}w}-1\right)=\left(-\frac{1}{3}w\right)^2-1^2=\frac{1}{9}w^2-1$$

27) $$\left[2(5n+1)\left(\frac{1}{5}n-\frac{1}{6}\right)-2\left(n-\frac{1}{2}\right)\left(n+\frac{1}{2}\right)+\frac{4}{15}n\right]^2-\frac{1}{9}\left(\frac{1}{4}-3n\right)=$$
$$=\left[2\left(n^2-\frac{5}{6}n+\frac{1}{5}n-\frac{1}{6}\right)-2\left(n^2-\frac{1}{4}\right)+\frac{4}{15}n\right]^2-\frac{1}{36}+\frac{1}{3}n=$$
$$=\left[\cancel{2n^2}\underline{-\frac{5}{3}n}\ \underline{+\frac{2}{5}n}\ \underline{\underline{-\frac{1}{3}}}\ \cancel{-2n^2}\ \underline{\underline{+\frac{1}{2}}}\ \underline{+\frac{4}{15}n}\right]^2-\frac{1}{36}+\frac{1}{3}n=\left[\left(-\frac{5}{3}+\frac{2}{5}+\frac{4}{15}\right)n-\frac{1}{3}+\frac{1}{2}\right]^2-\frac{1}{36}+\frac{1}{3}n=$$
$$=\left[\frac{-25+6+4}{15}n+\frac{-2+3}{6}\right]^2-\frac{1}{36}+\frac{1}{3}n=\left(\frac{\cancel{-15}}{\cancel{15}}n+\frac{1}{6}\right)^2-\frac{1}{36}+\frac{1}{3}n=n^2\ \cancel{-\frac{1}{3}n}\ \cancel{+\frac{1}{36}}\ \cancel{-\frac{1}{36}}\ \cancel{+\frac{1}{3}n}=n^2$$

28) $$a^2+(a+3)(-a-3)+3(2a+3)\underset{NOTA}{=}$$
$$=\cancel{a^2}\ \cancel{-a^2}\ \cancel{-3a}\ \cancel{-3a}\ \cancel{-9}\ \cancel{+6a}\ \cancel{+9}=0$$

NOTA
Anche:
$(a+3)(-a-3)=(a+3)[-(a+3)]=-(a+3)^2=ecc.$

39) $$(x-1)^2(x-2)^2-\left(x^2-3x+2\right)^2=[(x-1)(x-2)]^2-\left(x^2-3x+2\right)^2=$$
$$=\left(x^2-2x-x+2\right)^2-\left(x^2-3x+2\right)^2=\cancel{\left(x^2-3x+2\right)^2}\ \cancel{-\left(x^2-3x+2\right)^2}=0$$

decisamente più conveniente rispetto al possibile, ma noioso

$$(x-1)^2(x-2)^2-\left(x^2-3x+2\right)^2=\left(x^2-2x+1\right)\left(x^2-4x+4\right)-\left(x^4+9x^2+4-6x^3+4x^2-12x\right)=$$
$$=\underline{\cancel{x^4}}\ \underline{\underline{\cancel{-4x^3}}}\ \underset{*}{\cancel{+4x^2}}\ \underline{\underline{\cancel{-2x^3}}}\ \underset{*}{\cancel{+8x^2}}\ \underset{\square}{\cancel{-8x}}\ \underset{*}{\cancel{+x^2}}\ \underset{\square}{\cancel{-4x}}\ \cancel{+4}\ \underline{\cancel{-x^4}}\ \underset{*}{\cancel{-9x^2}}\ \cancel{-4}\ \underline{\underline{\cancel{+6x^3}}}\ \underset{*}{\cancel{-4x^2}}\ \underset{\square}{\cancel{+12x}}=0$$