

ESERCIZI CON ESPONENTI FRAZIONARI - SVOLGIMENTI

$$1) \quad 8^{\frac{2}{3}} + 9^{\frac{1}{2}} = \sqrt[3]{8^2} + \sqrt{9} = \sqrt[3]{64} + 3 = 4 + 3 = 7$$

$$2) \quad 4^{\frac{3}{2}} - 8^{\frac{1}{3}} - 3 \cdot 16^{\frac{1}{4}} = \sqrt{4^3} - \sqrt[3]{8} - 3\sqrt[4]{16} = \sqrt{64} - 2 - 3 \cdot 2 = 8 - 2 - 6 = 0$$

$$3) \quad \left[30\left(25^{-\frac{1}{2}} + 9^{-\frac{1}{2}}\right)\right]^{\frac{1}{4}} = \left[30\left(\frac{1}{\sqrt{25}} + \frac{1}{\sqrt{9}}\right)\right]^{\frac{1}{4}} = \sqrt[4]{30\left(\frac{1}{5} + \frac{1}{3}\right)} = \sqrt[4]{2 \cdot \frac{8}{15}} = \sqrt[4]{16} = 2$$

$$4) \quad \left(\frac{8}{27}\right)^{-\frac{1}{3}} - \left(\frac{1}{4}\right)^{\frac{1}{2}} = \left(\frac{27}{8}\right)^{\frac{1}{3}} - \sqrt{\frac{1}{4}} = \sqrt[3]{\frac{27}{8}} - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$$

$$5) \quad 64^{\frac{5}{6}} - 81^{\frac{3}{4}} = \sqrt[6]{64^5} - \sqrt[4]{81^3} = (\sqrt[6]{64})^5 - (\sqrt[4]{81})^3 = 2^5 - 3^3 = 32 - 27 = 5$$

$$6) \quad 2^{\frac{1}{2}} + 2^{-\frac{1}{2}} = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$7) \quad 3^{\frac{1}{4}} \cdot 27^{\frac{1}{4}} = (3 \cdot 27)^{\frac{1}{4}} = 81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$

$$8) \quad 5^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{6}} = 5^{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}} = 5^{\frac{3+2+1}{6}} = 5^{\frac{6}{6}} = 5$$

$$9) \quad \left(81^{\frac{1}{2}}\right)^{-\frac{1}{2}} = 81^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$$

$$10) \quad 2^{-\frac{2}{3}} \cdot 4^{-\frac{2}{3}} = (2 \cdot 4)^{-\frac{2}{3}} = 8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$$

$$11) \quad \sqrt{a} \cdot \sqrt[3]{a^2} \cdot \sqrt[4]{a^3} = a^{\frac{1}{2}} \cdot a^{\frac{2}{3}} \cdot a^{\frac{3}{4}} = a^{\frac{1}{2} + \frac{2}{3} + \frac{3}{4}} = a^{\frac{23}{12}} \quad 12) \quad \frac{\sqrt[4]{b^5}}{b \cdot \sqrt[3]{b}} = \frac{b^{\frac{5}{4}}}{b \cdot b^{\frac{1}{3}}} = \frac{b^{\frac{5}{4}}}{b^{1 + \frac{1}{3}}} = \frac{b^{\frac{5}{4}}}{b^{\frac{4}{3}}} = b^{\frac{5}{4} - \frac{4}{3}} = b^{\frac{5}{12}}$$

$$13) \quad \left(\frac{\sqrt{\sqrt{x^3}} \cdot \sqrt[3]{x}}{x}\right)^{12} = \left[\frac{\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}} \cdot x^{\frac{1}{3}}}{x}\right]^{12} = \left(\frac{x^{\frac{3}{4}} \cdot x^{\frac{1}{3}}}{x}\right)^{12} = \left(x^{\frac{3}{4} + \frac{1}{3} - 1}\right)^{12} = \left(x^{\frac{1}{12}}\right)^{12} = x^1 = x$$

$$14) \quad \sqrt[3]{\frac{\sqrt{10a} \cdot \sqrt[5]{a^2}}{\sqrt{a}}} = \left(\frac{a^{\frac{1}{10}} \cdot a^{\frac{2}{5}}}{a^{\frac{1}{2}}}\right)^{\frac{1}{3}} = \left(a^{\frac{1}{10} + \frac{2}{5} - \frac{1}{2}}\right)^{\frac{1}{3}} = \left(a^{\frac{1+4-5}{10}}\right)^{\frac{1}{3}} = (a^0)^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$$

$$15) \quad (\sqrt[4]{a} + \sqrt[4]{b})(\sqrt[4]{a} - \sqrt[4]{b}) = \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}} - b^{\frac{1}{4}}\right) = \left(a^{\frac{1}{4}}\right)^2 - \left(b^{\frac{1}{4}}\right)^2 = a^{\frac{1}{2}} - b^{\frac{1}{2}} = \sqrt{a} - \sqrt{b}$$

$$16) \quad \frac{y^2 \cdot \sqrt{3x} \cdot \sqrt[4]{9xy}}{\sqrt[3]{y^2}} = \frac{y^2 \cdot (3x)^{\frac{1}{2}} \cdot (9xy)^{\frac{1}{4}}}{y^{\frac{2}{3}}} = y^2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot 9^{\frac{1}{4}} \cdot x^{\frac{1}{4}} \cdot y^{\frac{1}{4}} \cdot y^{-\frac{2}{3}} = 3^{\frac{1}{2}} \cdot (3^2)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot y^2 \cdot y^{\frac{1}{4}} \cdot y^{-\frac{2}{3}} = 3^{\frac{1}{2} + \frac{1}{2}} \cdot x^{\frac{1}{2} + \frac{1}{4}} \cdot y^{2 + \frac{1}{4} - \frac{2}{3}} = 3x^{\frac{3}{4}} y^{\frac{19}{12}}$$

$$17) \quad \frac{\sqrt[3]{2\sqrt{2}}}{\sqrt[6]{\sqrt{2}+1} \cdot \sqrt[6]{\sqrt{2}-1}} = \frac{\left(2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}}}{\left(2^{\frac{1}{2}}+1\right)^{\frac{1}{6}} \cdot \left(2^{\frac{1}{2}}-1\right)^{\frac{1}{6}}} = \frac{\left(2^{1+\frac{1}{2}}\right)^{\frac{1}{3}}}{\left[\left(2^{\frac{1}{2}}+1\right) \cdot \left(2^{\frac{1}{2}}-1\right)\right]^{\frac{1}{6}}} = \frac{\left(2^{\frac{3}{2}}\right)^{\frac{1}{3}}}{\left[\left(2^{\frac{1}{2}}\right)^2 - 1\right]^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}}{\left[2^1 - 1\right]^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}}{1^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}}{1} = 2^{\frac{1}{2}} = \sqrt{2}$$

$$18) \quad \sqrt[13]{(\sqrt{t+1})^3 \cdot \sqrt[3]{(t+1)^2}} = \left[(t+1)^{\frac{3}{2}} \cdot (t+1)^{\frac{2}{3}}\right]^{\frac{1}{13}} = \left[(t+1)^{\frac{3+2}{2+3}}\right]^{\frac{1}{13}} = \left[(t+1)^{\frac{13}{5}}\right]^{\frac{1}{13}} = (t+1)^{\frac{1}{5}} = \sqrt[5]{t+1}$$