

n) ESPRESSIONI VARIE - SVOLGIMENTI

$$15) \sqrt[3]{\frac{t^2-4}{t-1}} \cdot \sqrt{\frac{t^2-3t+2}{t+2}} = \sqrt[3]{\frac{(t+2)(t-2)}{t-1}} \cdot \sqrt{\frac{(t-1)(t-2)}{t+2}} =$$

$$= \sqrt[6]{\frac{(t+2)^2(t-2)^2}{(t-1)^2}} \cdot \sqrt[6]{\frac{(t-1)^3(t-2)^3}{(t+2)^3}} = \sqrt[6]{\frac{(t+2)^2(t-2)^2}{\cancel{(t-1)^2} \cdot (t+2)^3} \cdot \frac{(t-1)^3(t-2)^3}{(t+2)^3}} = \sqrt[6]{\frac{(t-1)(t-2)^5}{t+2}}$$

$$16) \frac{\sqrt{2} \cdot \sqrt[3]{3}}{\sqrt[4]{4} \cdot \sqrt[5]{5}} = \frac{\sqrt{2} \cdot \sqrt[3]{3}}{\sqrt[2]{2} \cdot \sqrt[5]{5}} = \frac{\sqrt[2]{2} \cdot \sqrt[3]{3}}{\sqrt[2]{2} \cdot \sqrt[5]{5}} = \frac{15\sqrt[5]{3^5}}{15\sqrt[5]{5^3}} = \sqrt[5]{\frac{243}{125}} = \sqrt[5]{1,944} \approx 1,045$$

$$17) \sqrt{x\sqrt[3]{x}} : \sqrt[3]{x\sqrt{x}} = \sqrt{\sqrt[3]{x^4}} : \sqrt[3]{\sqrt{x^3}} = \sqrt[6]{x^4} : \sqrt[6]{x^3} = \sqrt[6]{x}$$

$$18) \frac{\left(\sqrt[3]{(x-y)^4} \sqrt{(x-y)}\right)^8}{x^3 - y^3} = \frac{\left(\sqrt[3]{4(x-y)^4(x-y)}\right)^8}{x^3 - y^3} = \frac{\left(\sqrt[12]{(x-y)^5}\right)^8}{x^3 - y^3} =$$

$$= \frac{\sqrt[3]{(x-y)^{10}}}{x^3 - y^3} = \frac{(x-y)^{\frac{10}{3}} \sqrt[3]{x-y}}{\cancel{(x-y)}(x^2 + xy + y^2)} = \frac{(x-y)^2 \sqrt[3]{x-y}}{x^2 + xy + y^2}$$

$$19) \sqrt[k]{a^p} \cdot \sqrt[p]{a^k} = \sqrt[kp]{a^{p^2}} \cdot \sqrt[p]{a^{k^2}} = \sqrt[kp]{a^{p^2} \cdot a^{k^2}} = \sqrt[kp]{a^{p^2+k^2}}$$

$$20) \frac{a}{b} \sqrt{\frac{b}{a}} \cdot \sqrt[4]{\frac{b}{a}} \cdot \sqrt[8]{ab} - \sqrt[8]{\frac{a^3}{b}} = \frac{a}{b} \sqrt[8]{\frac{b^4}{a^4} \cdot \frac{b^2}{a^2} \cdot ab} - \sqrt[8]{\frac{a^3}{b}} = \frac{a}{b} \sqrt[8]{\frac{b^7}{a^5}} - \sqrt[8]{\frac{a^3}{b}} =$$

$$= \sqrt[8]{\frac{a^8}{b^8} \cdot \frac{b^7}{a^5}} - \sqrt[8]{\frac{a^3}{b}} = \sqrt[8]{\frac{a^3}{b}} - \sqrt[8]{\frac{a^3}{b}} = 0$$

$$21) \sqrt{160} + \sqrt{250} + \sqrt{360} - \sqrt{2250} = \sqrt{16 \cdot 10} + \sqrt{25 \cdot 10} + \sqrt{36 \cdot 10} - \sqrt{225 \cdot 10} =$$

$$= 4\sqrt{10} + 5\sqrt{10} + 6\sqrt{10} - 15\sqrt{10} = 0$$

$$22) \frac{\sqrt{108} + \sqrt{243}}{\sqrt[3]{81} - 2\sqrt[3]{3}} = \frac{\sqrt{2^2 \cdot 3^3} + \sqrt{3^5}}{\sqrt[3]{3^4} - 2\sqrt[3]{3}} = \frac{2 \cdot 3\sqrt{3} + 3^2\sqrt{3}}{3\sqrt[3]{3} - 2\sqrt[3]{3}} = \frac{15\sqrt{3}}{\sqrt[3]{3}} = 15\sqrt[6]{\frac{3^3}{3^2}} = 15\sqrt[6]{3}$$

$$23) \sqrt{x^3 - 3x^2 + 3x - 1} + \sqrt{9x - 9} = \sqrt{(x-1)^3} + \sqrt{9(x-1)} = (x-1)\sqrt{x-1} + 3\sqrt{x-1} = (x+2)\sqrt{x-1}$$

$$24) \frac{\sqrt[4]{243} + \sqrt[4]{48}}{\sqrt{13^2 - 12^2}} = \frac{\sqrt[4]{3^5} + \sqrt[4]{2^4 \cdot 3}}{\sqrt{169 - 144}} = \frac{3\sqrt[4]{3} + 2\sqrt[4]{3}}{\sqrt{25}} = \frac{3\sqrt[4]{3} + 2\sqrt[4]{3}}{5} = \frac{5\sqrt[4]{3}}{5} = \sqrt[4]{3}$$

Occhio! GRAVE ERRORE
SAREBBE STATO scrivere
 $\sqrt{13^2 - 12^2} = \sqrt{13 - 12} = 1$!!!!

$$25) \sqrt{(2+\sqrt{3})^2 + (2\sqrt{3}-1)^2} = \sqrt{4+3+4\sqrt{3}+12+1-4\sqrt{3}} = \sqrt{20} = \boxed{2\sqrt{5}}$$

Occhio!!! SAREBBE STATO GRAVE ERRORE mandar via la radice coi due quadrati!

$$26) (\sqrt[4]{1+m} - \sqrt[4]{1-m})(\sqrt[4]{1+m} + \sqrt[4]{1-m})(\sqrt{1+m} + \sqrt{1-m}) =$$

$$= \left[\left(\sqrt[4]{1+m} \right)^2 - \left(\sqrt[4]{1-m} \right)^2 \right] (\sqrt{1+m} + \sqrt{1-m}) =$$

$$= (\sqrt{1+m} - \sqrt{1-m})(\sqrt{1+m} + \sqrt{1-m}) = (\sqrt{1+m})^2 - (\sqrt{1-m})^2 = (1+m) - (1-m) = 1+m-1+m = \boxed{2m}$$

$$27) \left(2\sqrt{2} - \sqrt{3} - 1 \right)^2 - \left(2\sqrt{3} - 3\sqrt{2} \right)^2 + 2\sqrt{2} \left(2 + \frac{9}{2}\sqrt{2} - 4\sqrt{3} - \sqrt{\frac{3}{2}} \right) =$$

$$= 8+3+1-4\sqrt{6}-4\sqrt{2}+2\sqrt{3} - (12-12\sqrt{6}+18) + 4\sqrt{2}+18-8\sqrt{6}-2\sqrt{3} =$$

$$= 30-12\sqrt{6}-12+12\sqrt{6}-18 = \boxed{0}$$

$$28) \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} - \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{(\sqrt{a} + \sqrt{b})^2 - (\sqrt{a} - \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{a+b+2\sqrt{ab} - (a+b-2\sqrt{ab})}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} =$$

$$= \frac{a+b+2\sqrt{ab} - a - b + 2\sqrt{ab}}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{4\sqrt{ab}}{a-b}$$

(In alternativa, si poteva procedere razionalizzando le singole frazioni...)

$$29) \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}} + \sqrt{\frac{x^2}{y^2} - 1} = \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}} \cdot \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} + \sqrt{\frac{x^2 - y^2}{y^2}} =$$

$$= \frac{(\sqrt{x+y} - \sqrt{x-y})^2}{x+y-(x-y)} + \frac{1}{y} \sqrt{x^2 - y^2} = \frac{x+y+x-y-2\sqrt{x^2-y^2}}{2y} + \frac{1}{y} \sqrt{x^2 - y^2} =$$

$$= \frac{2x-2\sqrt{x^2-y^2}}{2y} + \frac{1}{y} \sqrt{x^2-y^2} = \frac{\cancel{2} (x-\sqrt{x^2-y^2})}{\cancel{2} y} + \frac{\sqrt{x^2-y^2}}{y} = \frac{x-\sqrt{x^2-y^2} + \sqrt{x^2-y^2}}{y} = \frac{x}{y}$$

$$30) \frac{2\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} - \frac{x}{1-\sqrt{1-x^2}} = \frac{2\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} - \frac{x}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} =$$

$$= \frac{2\sqrt{1+x}(\sqrt{1+x} + \sqrt{1-x})}{1+x-(1-x)} - \frac{x(1+\sqrt{1-x^2})}{1-(1-x^2)} = \frac{\cancel{2} \sqrt{1+x}(\sqrt{1+x} + \sqrt{1-x})}{\cancel{2} x} - \frac{\cancel{x} (1+\sqrt{1-x^2})}{x\cancel{x}} =$$

$$= \frac{\cancel{1+x} + \sqrt{1-x^2} - \cancel{1} - \sqrt{1-x^2}}{x} = \frac{x}{x} = \boxed{1}$$

$$31) \frac{\frac{1}{1+\sqrt{x}} - 1}{(1+\sqrt[4]{x})(1-\sqrt[4]{x})} = \frac{1-1-\sqrt{x}}{1+\sqrt{x}} = -\frac{\sqrt{x}}{1+\sqrt{x}} \cdot \frac{1}{1-\sqrt{x}} = -\frac{\sqrt{x}}{1-x} = \frac{\sqrt{x}}{x-1}$$

$$\begin{aligned}
 32) \quad \frac{x^2 - x}{\sqrt{1-x^2} + x - 1} &= \frac{x(x-1)}{\sqrt{1-x^2} + x - 1} \cdot \frac{\sqrt{1-x^2} - (x-1)}{\sqrt{1-x^2} - (x-1)} = \frac{x(x-1) \left[\sqrt{1-x^2} - x + 1 \right]}{1-x^2 - (x-1)^2} = \\
 &= \frac{x(x-1) \left[\sqrt{1-x^2} - x + 1 \right]}{1-x^2 - x^2 + 2x - 1} = \frac{x(x-1) \left[\sqrt{1-x^2} - x + 1 \right]}{2x - 2x^2} = \frac{\cancel{x} \cdot \cancel{(x-1)} \left[\sqrt{1-x^2} - x + 1 \right]}{2 \cancel{x} (1-x)} = \\
 &= -\frac{\sqrt{1-x^2} - x + 1}{2} = \boxed{\frac{x-1-\sqrt{1-x^2}}{2}}
 \end{aligned}$$

$$\begin{aligned}
 33) \quad \frac{\sqrt{2}}{2\sqrt{2}+3} + \frac{\sqrt{3}}{2-\sqrt{3}} + 1 &= \frac{\sqrt{2}}{2\sqrt{2}+3} \cdot \frac{2\sqrt{2}-3}{2\sqrt{2}-3} + \frac{\sqrt{3}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} + 1 = \frac{\sqrt{2}(2\sqrt{2}-3)}{8-9} + \frac{\sqrt{3}(2+\sqrt{3})}{4-3} + 1 = \\
 &= -\sqrt{2}(2\sqrt{2}-3) + \sqrt{3}(2+\sqrt{3}) + 1 = \cancel{4} + 3\sqrt{2} + 2\sqrt{3} + \cancel{3} + 1 = \boxed{3\sqrt{2} + 2\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 34) \quad \sqrt[3]{2\sqrt{7}-1} \cdot \sqrt[3]{2\sqrt{7}+1} - \sqrt{16-6\sqrt{7}} - (\sqrt{\sqrt{7}})^2 &= \sqrt[3]{(2\sqrt{7}-1)(2\sqrt{7}+1)} - \sqrt{(3-\sqrt{7})^2} - \sqrt{7} = \\
 &= \sqrt[3]{(2\sqrt{7})^2 - 1^2} - (3-\sqrt{7}) - \sqrt{7} = \sqrt[3]{28-1} - 3 + \sqrt{7} - \sqrt{7} = \sqrt[3]{27} - 3 = 3 - 3 = \boxed{0}
 \end{aligned}$$

$$35) \quad \frac{\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}}{\sqrt{2\sqrt{3}+3} \cdot \sqrt{2\sqrt{3}-3}} = \frac{\sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{3}-1)^2}}{\sqrt{(2\sqrt{3}+3)(2\sqrt{3}-3)}} = \frac{\sqrt{3}+1 + \sqrt{3}-1}{\sqrt{12-9}} = \frac{2\sqrt{3}}{\sqrt{3}} = \boxed{2}$$

$$\begin{aligned}
 36) \quad \frac{\text{Scomposizione con Ruffini}}{\sqrt{x^3-3x-2} - \sqrt{x^3-6x^2+12x-8}} &= \frac{\sqrt{(x+1)^2(x-2)} - \sqrt{(x-2)^3}}{3} = \\
 &= \frac{(x+1)\sqrt{x-2} - (x-2)\sqrt{x-2}}{3} = \frac{(x+1-x+2)\sqrt{x-2}}{3} = \frac{3\sqrt{x-2}}{3} = \boxed{\sqrt{x-2}}
 \end{aligned}$$

$$\begin{aligned}
 37) \quad \left(\sqrt[4]{a} + \frac{\sqrt[3]{a^2}}{\sqrt{a\sqrt{a}}} \right) \left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}} \right) + \sqrt[3]{a} &= \left(\sqrt[4]{a} + \frac{\sqrt[3]{a^2}}{\sqrt{\sqrt{a^3}}} \right) \left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}} \right) + \sqrt[3]{a} = \\
 &= \left(\sqrt[4]{a} + \frac{\sqrt[3]{a^2}}{\sqrt[4]{a^3}} \right) \left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}} \right) + \sqrt[3]{a} = \left(\frac{\sqrt[4]{a^4} + \sqrt[3]{a^2}}{\sqrt[4]{a^3}} \right) \left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}} \right) + \sqrt[3]{a} = \frac{a + \sqrt[3]{a^2}}{\sqrt[4]{a^3}} \cdot \frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}} + \sqrt[3]{a} = \\
 &= \frac{a^2 - \sqrt[3]{a^4}}{\sqrt[4]{a^4}} + \sqrt[3]{a} = \frac{a^2 - \sqrt[3]{a^4}}{a} + \sqrt[3]{a} = a - \sqrt[3]{a} + \sqrt[3]{a} = \boxed{a}
 \end{aligned}$$

$$38) \quad \sqrt{\frac{3}{4}} + \frac{3}{4}\sqrt{\frac{4}{3}} = \sqrt{\frac{3}{4}} + \sqrt{\frac{9^3}{16^4} \cdot \frac{4}{3}} = \sqrt{\frac{3}{4}} + \sqrt{\frac{3}{4}} = 2\sqrt{\frac{3}{4}} = \sqrt{4 \cdot \frac{3}{4}} = \boxed{\sqrt{3}}$$

oppure:

$$\sqrt{\frac{3}{4}} + \frac{3}{4}\sqrt{\frac{4}{3}} = \frac{\sqrt{3}}{\sqrt{4}} + \frac{3}{4} \cdot \frac{\sqrt{4}}{\sqrt{3}} = \frac{\sqrt{3}}{2} + \frac{3}{4} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3} \cdot \cancel{3}}{2\sqrt{3}} = \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \boxed{\sqrt{3}}$$

$$39) \frac{a\sqrt[3]{a} + \sqrt[3]{a} - \sqrt[3]{a}}{a\sqrt[3]{b} + \sqrt[3]{b} - \sqrt[3]{b}} = \frac{\cancel{(a+1)}\sqrt[3]{a}}{\cancel{(a+1)}\sqrt[3]{b}} - \sqrt[3]{a} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \boxed{0}$$

$$40) \frac{1}{4} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{4} = \frac{1}{4} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{4} =$$

$$= \frac{1}{4} \cdot \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} + \frac{\sqrt{6}}{2} - \frac{1}{4} =$$

$$= \frac{1}{4} \cdot (3+2-2\sqrt{6}) + \frac{\sqrt{6}}{2} - \frac{1}{4} = \frac{5}{4} - \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2} - \frac{1}{4} = \frac{4}{4} = \boxed{1}$$

$$41) \frac{(\sqrt{3}+1)^2 - (\sqrt{2}-1)^2 - 1}{\sqrt{12} + \sqrt{8}} = \frac{\cancel{3} + \cancel{2}\sqrt{3} - \cancel{2}\sqrt{2} + \cancel{1} + 2\sqrt{2} - \cancel{1}}{2\sqrt{3} + 2\sqrt{2}} = \frac{2\sqrt{3} + 2\sqrt{2}}{2\sqrt{3} + 2\sqrt{2}} = \boxed{1}$$

$$42) \frac{\sqrt{\sqrt{9+4\sqrt{2}} + \sqrt{9-4\sqrt{2}}}}{2} = \frac{\sqrt{\sqrt{(2\sqrt{2}+1)^2} + \sqrt{(2\sqrt{2}-1)^2}}}{2} =$$

$$= \frac{\sqrt{2\sqrt{2} + 1 + 2\sqrt{2} - 1}}{2} = \frac{\sqrt{4\sqrt{2}}}{2} = \frac{\cancel{2}\sqrt{\sqrt{2}}}{\cancel{2}} = \boxed{\sqrt[4]{2}}$$

$$43) \left(\frac{\sqrt[4]{x}}{2} - \frac{2}{\sqrt[4]{x}} \right) : \sqrt[4]{x} + 2 \frac{\sqrt{x}}{x} = \frac{\sqrt{x}-4}{2\sqrt[4]{x}} : \sqrt[4]{x} + \frac{2\sqrt{x}}{x} = \frac{\sqrt{x}-4}{2\sqrt[4]{x}} \cdot \frac{1}{\sqrt[4]{x}} + \frac{2\sqrt{x}}{x} =$$

$$= \frac{\sqrt{x}-4}{2\sqrt{x}} + \frac{2\sqrt{x}}{x} = \frac{\sqrt{x}(\sqrt{x}-4) + 4\sqrt{x}}{2x} =$$

$$= \frac{\cancel{x} - \cancel{4}\sqrt{x} + 4\sqrt{x}}{2x} = \frac{x}{2x} = \boxed{\frac{1}{2}}$$

$$44) \left(\frac{1}{\sqrt{1-w}} + \frac{1}{\sqrt{1+w}} \right)^2 - \frac{2}{\sqrt{1-w^2}} = \left(\frac{\sqrt{1+w} + \sqrt{1-w}}{\sqrt{1-w} \cdot \sqrt{1+w}} \right)^2 - \frac{2}{\sqrt{1-w^2}} \cdot \frac{\sqrt{1-w^2}}{\sqrt{1-w^2}} =$$

$$= \frac{\cancel{1+w} + \cancel{1-w} + 2\sqrt{1-w^2}}{(1-w)(1+w)} - \frac{2\sqrt{1-w^2}}{1-w^2} =$$

$$= \frac{2+2\sqrt{1-w^2}}{1-w^2} - \frac{2\sqrt{1-w^2}}{1-w^2} =$$

$$= \frac{\cancel{2} + 2\sqrt{1-w^2} - \cancel{2}\sqrt{1-w^2}}{1-w^2} = \boxed{\frac{2}{1-w^2}}$$

$$45) \frac{\sqrt{1+x}}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{1+x}} - \left(\frac{1}{\sqrt{x}} - \sqrt{x} \right) : \sqrt{1+x} = \frac{1+x-2x}{\sqrt{x} \cdot \sqrt{1+x}} - \frac{1-x}{\sqrt{x}} \cdot \frac{1}{\sqrt{1+x}} = \frac{1-x}{\sqrt{x} \cdot \sqrt{1+x}} - \frac{1-x}{\sqrt{x} \cdot \sqrt{1+x}} = \boxed{0}$$

$$46) \frac{\sqrt[3]{\frac{a}{b}} \cdot \sqrt[4]{\frac{b}{a}}}{\sqrt{\sqrt[3]{\frac{a}{b}}}} = \frac{\sqrt[12]{\frac{a^4}{b^4}} \cdot \sqrt[12]{\frac{b^3}{a^3}}}{\sqrt[12]{\frac{a}{b}}} = \frac{\sqrt[12]{\frac{a^4}{b^4} \cdot \frac{b^3}{a^3}}}{\sqrt[12]{\frac{a}{b}}} = \frac{\sqrt[12]{\frac{a}{b}}}{\sqrt[12]{\frac{a}{b}}} = \boxed{1}$$

$$47) \frac{(\sqrt{a})^4 - (\sqrt{b})^4}{b \left(\frac{\sqrt[3]{a}}{\sqrt{b}} \right)^6 - 1} = \frac{a^2 - b^2}{b \cdot \frac{a^2}{b^{\frac{6}{2}}} - 1} = \frac{a^2 - b^2}{\frac{a^2 - b^2}{b^2}} = \sqrt{\frac{(a^2 - b^2) \cdot b^2}{a^2 - b^2}} = \sqrt{b^2} = \boxed{b}$$

$$48) \frac{1}{x + \sqrt[3]{1-x^3}} = \frac{1}{x + \sqrt[3]{1-x^3}} \cdot \frac{x^2 - x\sqrt[3]{1-x^3} + \sqrt[3]{(1-x^3)^2}}{x^2 - x\sqrt[3]{1-x^3} + \sqrt[3]{(1-x^3)^2}} =$$

$$= \frac{x^2 - x\sqrt[3]{1-x^3} + \sqrt[3]{(1-x^3)^2}}{x^3 + (\sqrt[3]{1-x^3})^3} =$$

$$= \frac{x^2 - x\sqrt[3]{1-x^3} + \sqrt[3]{(1-x^3)^2}}{x^3 + 1 - x^3} =$$

$$\boxed{\frac{x^2 - x\sqrt[3]{1-x^3} + \sqrt[3]{(1-x^3)^2}}{x^3 + 1 - x^3}}$$

$$49) \frac{2}{\sqrt[4]{a+1} + \sqrt[4]{a-1}} = \frac{2}{\sqrt[4]{a+1} + \sqrt[4]{a-1}} \cdot \frac{\sqrt[4]{a+1} - \sqrt[4]{a-1}}{\sqrt[4]{a+1} - \sqrt[4]{a-1}} = \frac{2(\sqrt[4]{a+1} - \sqrt[4]{a-1})}{(2\sqrt[4]{a+1})^2 - (2\sqrt[4]{a-1})^2} =$$

$$= \frac{2(\sqrt[4]{a+1} - \sqrt[4]{a-1}) \cdot \sqrt{a+1} + \sqrt{a-1}}{\sqrt{a+1} - \sqrt{a-1}} = \frac{2(\sqrt[4]{a+1} - \sqrt[4]{a-1})(\sqrt{a+1} + \sqrt{a-1})}{a+1 - a+1} =$$

$$= \frac{\cancel{2}(\sqrt[4]{a+1} - \sqrt[4]{a-1})(\sqrt{a+1} + \sqrt{a-1})}{\cancel{2}} = \boxed{(\sqrt[4]{a+1} - \sqrt[4]{a-1})(\sqrt{a+1} + \sqrt{a-1})}$$

$$50) \frac{\sqrt{ac} - \sqrt{c}}{a - \sqrt{a} + \sqrt{ab} - \sqrt{b}} = \frac{\sqrt{c}(\sqrt{a} - 1)}{\sqrt{a}(\sqrt{a} - 1) + \sqrt{b}(\sqrt{a} - 1)} = \frac{\sqrt{c}(\sqrt{a} - 1)}{(\sqrt{a} - 1)(\sqrt{a} + \sqrt{b})} = \boxed{\frac{\sqrt{c}}{\sqrt{a} + \sqrt{b}}}$$

$$51) \frac{\sqrt{10} + \sqrt{6} - \sqrt{5} - \sqrt{3}}{2\sqrt{6} - 2\sqrt{3} - \sqrt{2} + 1} = \frac{\sqrt{2}(\sqrt{5} + \sqrt{3}) - (\sqrt{5} + \sqrt{3})}{2\sqrt{3}(\sqrt{2} - 1) - (\sqrt{2} - 1)} = \frac{(\sqrt{5} + \sqrt{3})(\sqrt{2} - 1)}{(\sqrt{2} - 1)(2\sqrt{3} - 1)} = \boxed{\frac{\sqrt{5} + \sqrt{3}}{2\sqrt{3} - 1}}$$

$$52) \frac{\sqrt{3} + \sqrt{2} + 1}{\sqrt{3} - \sqrt{2} + 1} = \frac{\sqrt{3} + \sqrt{2} + 1}{\sqrt{3} - (\sqrt{2} - 1)} \cdot \frac{\sqrt{3} + (\sqrt{2} - 1)}{\sqrt{3} + (\sqrt{2} - 1)} = \frac{(\sqrt{3} + \sqrt{2} + 1)(\sqrt{3} + \sqrt{2} - 1)}{(\sqrt{3})^2 - (\sqrt{2} - 1)^2} =$$

$$= \frac{(\sqrt{3} + \sqrt{2})^2 - 1^2}{3 - (2 + 1 - 2\sqrt{2})} = \frac{3 + 2 + 2\sqrt{6} - 1}{2\sqrt{2}} = \frac{4 + 2\sqrt{6}}{2\sqrt{2}} = \frac{\cancel{2}(2 + \sqrt{6})}{\cancel{2}\sqrt{2}} = \frac{2 + \sqrt{6}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} =$$

$$= \frac{2\sqrt{2} + \sqrt{12}}{2} = \frac{2\sqrt{2} + 2\sqrt{3}}{2} = \boxed{\sqrt{2} + \sqrt{3} \approx 1,414 + 1,732 = 3,146}$$