l) RAZIONALIZZAZIONE - SVOLGIMENTI

76) $\frac{x}{2\sqrt{x}} = \frac{x}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{\cancel{x}\sqrt{x}}{2\cancel{x}} = \boxed{\frac{\sqrt{x}}{2}}$ o anche: $\frac{x}{2\sqrt{x}} = \frac{\sqrt{x}\cdot\cancel{\sqrt{x}}}{2\cancel{\sqrt{x}}} = \boxed{\frac{\sqrt{x}}{2}}$

77) $\frac{9-4t}{3-2\sqrt{t}} = \frac{9-4t}{3-2\sqrt{t}}\cdot\frac{3+2\sqrt{t}}{3+2\sqrt{t}} = \frac{\cancel{(9-4t)}\left(3+2\sqrt{t}\right)}{\cancel{9-4t}} = \boxed{3+2\sqrt{t}}$

o anche: $\frac{9-4t}{3-2\sqrt{t}} = \frac{\left(3+2\sqrt{t}\right)\cancel{\left(3-2\sqrt{t}\right)}}{\cancel{3-2\sqrt{t}}} = \boxed{3+2\sqrt{t}}$

78) $\frac{6}{3\sqrt{2}-2\sqrt{3}} = \frac{6}{3\sqrt{2}-2\sqrt{3}}\cdot\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{6\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2} = \frac{\cancel{6}\left(3\sqrt{2}+2\sqrt{3}\right)}{\cancel{18-12}} = \boxed{3\sqrt{2}+2\sqrt{3}}$

79) $\frac{2\sqrt{x}-1}{2\sqrt{x}+1} = \frac{2\sqrt{x}-1}{2\sqrt{x}+1}\cdot\frac{2\sqrt{x}-1}{2\sqrt{x}-1} = \frac{\left(2\sqrt{x}-1\right)^2}{\left(2\sqrt{x}\right)^2-1^2} = \boxed{\frac{4x+1-4\sqrt{x}}{4x-1}}$

80) $\frac{y}{\sqrt{x}-\sqrt{x+y}} = \frac{y}{\sqrt{x}-\sqrt{x+y}}\cdot\frac{\sqrt{x}+\sqrt{x+y}}{\sqrt{x}+\sqrt{x+y}} = \frac{y\left(\sqrt{x}+\sqrt{x+y}\right)}{\cancel{x}-\cancel{x}-y} = \boxed{-\left(\sqrt{x}+\sqrt{x+y}\right)}$

81) $\frac{a^2-1}{\sqrt{a+1}} = \frac{a^2-1}{\sqrt{a+1}}\cdot\frac{\sqrt{a+1}}{\sqrt{a+1}} = \frac{\cancel{(a+1)}\,(a-1)\sqrt{a+1}}{\cancel{a+1}} = \boxed{(a-1)\sqrt{a+1}}$

82) $\frac{2c-18}{3\sqrt{c}+9} = \frac{2(c-9)}{3\left(\sqrt{c}+3\right)}\cdot\frac{\sqrt{c}-3}{\sqrt{c}-3} = \frac{2\cancel{(c-9)}\left(\sqrt{c}-3\right)}{3\cancel{(c-9)}} = \boxed{\frac{2\left(\sqrt{c}-3\right)}{3}\ \textit{oppure}\ \frac{2}{3}\left(\sqrt{c}-3\right)}$

83) $\frac{1}{\sqrt{3-\sqrt{5}}} = \frac{1}{\sqrt{3-\sqrt{5}}}\cdot\frac{\sqrt{3+\sqrt{5}}}{\sqrt{3+\sqrt{5}}} = \frac{\sqrt{3+\sqrt{5}}}{\sqrt{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}} = \frac{\sqrt{3+\sqrt{5}}}{\sqrt{9-5}} = \frac{\sqrt{3+\sqrt{5}}}{\sqrt{4}} = \boxed{\frac{\sqrt{3+\sqrt{5}}}{2}}$

84) $\frac{1}{\sqrt{5}+\sqrt{3}-1} = \frac{1}{\sqrt{5}+\left(\sqrt{3}-1\right)}\cdot\frac{\sqrt{5}-\left(\sqrt{3}-1\right)}{\sqrt{5}-\left(\sqrt{3}-1\right)} \underset{NOTA\,1}{=} \frac{\sqrt{5}-\sqrt{3}+1}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}-1\right)^2} =$

$= \frac{\sqrt{5}-\sqrt{3}+1}{5-\left(3+1-2\sqrt{3}\right)} = \frac{\sqrt{5}-\sqrt{3}+1}{5-4+2\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}+1}{1+2\sqrt{3}} \underset{NOTA\,2}{=}$

$= \frac{\sqrt{5}-\sqrt{3}+1}{2\sqrt{3}+1}\cdot\frac{2\sqrt{3}-1}{2\sqrt{3}-1} = \frac{2\sqrt{15}-\sqrt{5}-6+\sqrt{3}+2\sqrt{3}-1}{\left(2\sqrt{3}\right)^2-1^2} =$

$= \frac{2\sqrt{15}-\sqrt{5}+3\sqrt{3}-7}{12-1} = \boxed{\frac{2\sqrt{15}-\sqrt{5}+3\sqrt{3}-7}{11}}$

NOTA 1
Abbiamo scelto
questo raggruppamento
perché così i calcoli
si prospettano più semplici

NOTA 2
Invertiamo l'ordine,
affinché,
dopo la razionalizzazione,
si ottenga
un denominatore positivo

85) $\frac{\sqrt{60}}{\sqrt{5}-2\sqrt{2}+\sqrt{3}} = \frac{2\sqrt{15}}{\left(\sqrt{5}+\sqrt{3}\right)-2\sqrt{2}}\cdot\frac{\left(\sqrt{5}+\sqrt{3}\right)+2\sqrt{2}}{\left(\sqrt{5}+\sqrt{3}\right)+2\sqrt{2}} = \frac{2\sqrt{15}\left(\sqrt{5}+\sqrt{3}+2\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{3}\right)^2-\left(2\sqrt{2}\right)^2} =$

$= \frac{2\sqrt{15}\left(\sqrt{5}+\sqrt{3}+2\sqrt{2}\right)}{\cancel{5}+\cancel{3}+2\sqrt{15}-\cancel{8}} = \boxed{\sqrt{5}+\sqrt{3}+2\sqrt{2}}$

86) $$\frac{1}{2\sqrt[3]{2}-\sqrt[3]{15}}=\underbrace{\frac{1}{2\sqrt[3]{2}-\sqrt[3]{15}}}_{a-b}\cdot\frac{\left(2\sqrt[3]{2}\right)^2+2\sqrt[3]{2}\cdot\sqrt[3]{15}+\left(\sqrt[3]{15}\right)^2}{\underbrace{\left(2\sqrt[3]{2}\right)^2+2\sqrt[3]{2}\cdot\sqrt[3]{15}+\left(\sqrt[3]{15}\right)^2}_{a^2+ab+b^2}}=$$

$$=\frac{4\sqrt[3]{4}+2\sqrt[3]{30}+\sqrt[3]{225}}{\underbrace{\left(2\sqrt[3]{2}\right)^3-\left(\sqrt[3]{15}\right)^3}_{a^3-b^3}}=\frac{4\sqrt[3]{4}+2\sqrt[3]{30}+\sqrt[3]{225}}{16-15}=\boxed{4\sqrt[3]{4}+2\sqrt[3]{30}+\sqrt[3]{225}}$$

87) $$\frac{x^2-64}{\sqrt[3]{x}+2}=\frac{x^2-64}{\sqrt[3]{x}+2}\cdot\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}=\frac{\cancel{(x+8)}\,(x-8)\left(\sqrt[3]{x^2}-2\sqrt[3]{x}+4\right)}{\cancel{x+8}}=\boxed{(x-8)\left(\sqrt[3]{x^2}-2\sqrt[3]{x}+4\right)}$$

88) $$\frac{3}{2\sqrt[3]{3}}=\frac{3}{2\sqrt[3]{3}}\cdot\frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}=\frac{\cancel{3}\sqrt[3]{3^2}}{2\cdot\cancel{3}}=\boxed{\frac{\sqrt[3]{9}}{2}}$$

o anche:

$$\frac{3}{2\sqrt[3]{3}}=\frac{\sqrt[3]{3}\cdot\sqrt[3]{3}\cdot\cancel{\sqrt[3]{3}}}{2\cancel{\sqrt[3]{3}}}=\boxed{\frac{\sqrt[3]{9}}{2}}$$

89) $$\frac{a-b}{\sqrt[4]{(a-b)^3}}=\frac{a-b}{\sqrt[4]{(a-b)^3}}\cdot\frac{\sqrt[4]{a-b}}{\sqrt[4]{a-b}}=\frac{(a-b)\sqrt[4]{a-b}}{\sqrt[4]{(a-b)^4}}=\frac{\cancel{(a-b)}\sqrt[4]{a-b}}{\cancel{a-b}}=\boxed{\sqrt[4]{a-b}}$$

oppure: $$\frac{a-b}{\sqrt[4]{(a-b)^3}}=\frac{\left(\sqrt[4]{a-b}\right)^{\cancel{4}}}{\cancel{\left(\sqrt[4]{a-b}\right)^3}}=\boxed{\sqrt[4]{a-b}}$$