

ESERCIZI CONCLUSIVI SUI SISTEMI - CORREZIONI (numeri dispari)

9)

$$\begin{cases} \frac{x}{4} = \frac{y}{3} \\ x^2 - y^2 = 7 \end{cases} \quad \begin{cases} x = \frac{4}{3}y \\ x^2 - y^2 = 7 \end{cases} \quad \begin{cases} x = \frac{4}{3}y \\ \left(\frac{4}{3}y\right)^2 - y^2 = 7 \\ \frac{16}{9}y^2 - y^2 = 7 \\ \frac{7}{9}y^2 = 7 \quad \frac{1}{9}y^2 = 1 \quad y^2 = 9 \quad y = \pm 3 \end{cases}$$

$$\begin{cases} x = \frac{4}{3}y = -4 \\ y = -3 \end{cases} \vee \begin{cases} x = \frac{4}{3}y = 4 \\ y = 3 \end{cases}$$

11)

$$\begin{cases} a + b = 0 \\ a^2 + 2ab + 3b^2 = 8 \end{cases} \quad \begin{cases} b = -a \\ a^2 - 2a^2 + 3a^2 = 8; 2a^2 = 8; a^2 = 4; a = \pm 2 \end{cases} \quad \begin{cases} a = \pm 2 \\ b = -a = \mp 2 \end{cases}$$

13)

$$\begin{cases} u(v+1) + 1 = v^2 \\ 2v - u = 3 \end{cases}$$

$$\begin{cases} uv + u + 1 = v^2 \\ u = 2v - 3 \end{cases}$$

$$\begin{cases} u = 2v - 3 \\ (2v - 3)v + 2v - 3 + 1 = v^2 \\ 2v^2 - 3v + 2v - 2 = v^2 \\ v^2 - v - 2 = 0 \\ (v - 2)(v + 1) = 0 \\ v = 2 \vee v = -1 \end{cases}$$

$$\begin{cases} u = 2v - 3 = 4 - 3 = 1 \\ v = 2 \end{cases} \vee \begin{cases} u = 2v - 3 = -2 - 3 = -5 \\ v = -1 \end{cases}$$

15)

$$\begin{cases} x = y + 1 \\ x^2 = y^3 + 1 \end{cases} \quad \begin{cases} x = y + 1 \\ (y + 1)^2 = y^3 + 1 \end{cases}$$

$$y^2 + 2y \cancel{+ 1} = y^3 \cancel{+ 1}$$

$$y^3 - y^2 - 2y = 0$$

$$y(y^2 - y - 2) = 0$$

$$y(y - 2)(y + 1) = 0$$

$$y = 0 \vee y = 2 \vee y = -1$$

$$\begin{cases} x = y + 1 = 1 \\ y = 0 \end{cases} \vee \begin{cases} x = y + 1 = 3 \\ y = 2 \end{cases} \vee \begin{cases} x = y + 1 = 0 \\ y = -1 \end{cases}$$

17)

$$\begin{cases} x + y = 2\sqrt{5} \\ xy = 2 \end{cases}$$

$$t^2 - 2\sqrt{5}t + 2 = 0$$

$$t = \sqrt{5} \pm \sqrt{5 - 2} = \sqrt{5} \pm \sqrt{3}$$

$$\begin{cases} x = \sqrt{5} \pm \sqrt{3} \\ y = \sqrt{5} \mp \sqrt{3} \end{cases}$$

19)

$$\begin{cases} x + y = xy - 56 \\ xy = 72 \end{cases}$$

$$\begin{cases} x + y = 72 - 56 = 16 \\ xy = 72 \end{cases}$$

$$t^2 - 16t + 72 = 0$$

$$t_{1,2} = 8 \pm \sqrt{64 - 72} \text{ imp}$$

21)

$$\begin{cases} x^2 + y^2 = 20 \\ x + y = 4\sqrt{2} \end{cases} \quad \begin{cases} x + y = 4\sqrt{2} \\ (x + y)^2 - 2xy = 20 \end{cases}$$

$$\begin{cases} x + y = 4\sqrt{2} \\ (4\sqrt{2})^2 - 2xy = 20; 32 - 2xy = 20; -2xy = -12; xy = 6 \end{cases}$$

$$t^2 - 4t\sqrt{2} + 6 = 0$$

$$t_{1,2} = 2\sqrt{2} \pm \sqrt{8 - 6} = 2\sqrt{2} \pm \sqrt{2} = \begin{cases} \sqrt{2} \\ 3\sqrt{2} \end{cases}$$

$$\begin{cases} x = \sqrt{2} \\ y = 3\sqrt{2} \end{cases} \vee \begin{cases} x = 3\sqrt{2} \\ y = \sqrt{2} \end{cases}$$

23)

$$\begin{cases} x + y = 1 \\ x^3 + y^3 = 1 \end{cases} \quad \begin{cases} x + y = 1 \\ (x + y)^3 - 3xy(x + y) = 1 \end{cases} \quad \begin{cases} x + y = 1 \\ \cancel{1} - 3xy = \cancel{1} \end{cases}$$

$$\begin{cases} x + y = 1 \\ xy = 0; x = 0 \vee y = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 1 \end{cases} \vee \begin{cases} x = 1 \\ y = 0 \end{cases}$$

25)

$$\begin{cases} a + b + c = 3 \\ a - c = 1 \\ ab + ac + bc = 2 \end{cases}$$

$$\begin{cases} a = c + 1 \\ c + 1 + b + c = 3; b + 2c = 2; b = 2 - 2c \\ ab + ac + bc = 2 \end{cases}$$

$$\begin{cases} a = c + 1 \\ b = 2 - 2c \\ (c + 1)(2 - 2c) + (c + 1)c + (2 - 2c)c = 2 \end{cases}$$

$$\cancel{2c} - 2c^2 + \cancel{2c} + c^2 + c + 2c - 2c^2 = \cancel{2}$$

$$-3c^2 + 3c = 0$$

$$c^2 - c = 0$$

$$c(c - 1) = 0$$

$$c = 0 \vee c = 1$$

$$\begin{cases} a = c + 1 = 1 \\ b = 2 - 2c = 2 \\ c = 0 \end{cases} \vee \begin{cases} a = c + 1 = 1 + 1 = 2 \\ b = 2 - 2c = 2 - 2 = 0 \\ c = 1 \end{cases}$$

27)

$$\begin{cases} ab+1=0 \\ ac+2=0 \\ bc=a+b+c \end{cases} \begin{cases} b = -\frac{1}{a} \quad (a \neq 0) \\ c = -\frac{2}{a} \quad (a \neq 0) \\ -\frac{1}{a} \left(-\frac{2}{a} \right) = a - \frac{1}{a} - \frac{2}{a} \end{cases}$$

$$\frac{2}{a^2} = a - \frac{3}{a}$$

$$2 = a^3 - 3a$$

$$a^3 - 3a - 2 = 0$$

Ruffini: $(a+1)(a^2 - a - 2) = 0$; $(a+1)(a+1)(a-2) = 0$; $a = -1 \vee a = 2$

$$\begin{cases} a = -1 \\ b = -\frac{1}{a} = 1 \\ c = -\frac{2}{a} = 2 \end{cases} \vee \begin{cases} a = 2 \\ b = -\frac{1}{a} = -\frac{1}{2} \\ c = -\frac{2}{a} = -1 \end{cases}$$

29)

$$\begin{cases} (x+y)^2 = 6xy - x - y \\ 2x + 2y = 3xy \end{cases}$$

$$\begin{cases} (x+y)^2 = 6xy - (x+y) \\ 2(x+y) = 3xy \end{cases}$$

$$x+y = s, \quad xy = p$$

$$\begin{cases} s^2 = 6p - s \\ 2s = 3p \end{cases}$$

$$\begin{cases} p = \frac{2}{3}s \end{cases}$$

$$s^2 = 6 \cdot \frac{2}{3}s - s; \quad s^2 = 4s - s; \quad s^2 = 3s; \quad s = 0 \vee s = 3$$

$$\begin{cases} s = 0 \\ p = \frac{2}{3}s = 0 \end{cases} \vee \begin{cases} s = 3 \\ p = \frac{2}{3}s = 2 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 0 \end{cases} \vee \begin{cases} x = 1 \\ y = 2 \end{cases} \vee \begin{cases} x = 2 \\ y = 1 \end{cases}$$

31)

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = 6 \\ \frac{1}{x^2} + \frac{1}{y^2} = 18 \end{cases} \quad \frac{1}{x} = u \quad \frac{1}{y} = v \quad \left(x = \frac{1}{u}, \quad y = \frac{1}{v} \right)$$

$$\begin{cases} u+v = 6 \\ u^2 + v^2 = 18 \end{cases} \quad \begin{cases} u+v = 6 \\ (u+v)^2 - 2uv = 18 \end{cases} \quad 36 - 2uv = 18; \quad -2uv = -18; \quad uv = 9$$

$$\begin{cases} u+v = 6 \\ uv = 9 \end{cases} \quad t^2 - 6t + 9 = 0 \quad t_{1,2} = 3 \pm \sqrt{9-9} = 3$$

$$\begin{cases} u = 3 \\ v = 3 \end{cases} \rightarrow \begin{cases} x = \frac{1}{u} = \frac{1}{3} \\ y = \frac{1}{v} = \frac{1}{3} \end{cases}$$

33)

$$\begin{cases} xy + x = 35 \\ xy + y = 36 \end{cases}$$

$$(2) - (1) \begin{cases} y - x = 1 \end{cases}$$

$$(1) \begin{cases} xy + x = 35 \end{cases}$$

$$\begin{cases} y = x + 1 \end{cases}$$

$$\begin{cases} x(x+1) + x = 35; x^2 + x + x = 35; x^2 + 2x - 35 = 0; (x+7)(x-5) = 0; x = -7 \vee x = 5 \end{cases}$$

$$\begin{cases} x = -7 \\ y = x + 1 = -7 + 1 = -6 \end{cases} \vee \begin{cases} x = 5 \\ y = x + 1 = 5 + 1 = 6 \end{cases}$$

35)

$$\begin{cases} 3x - y = 13 \end{cases}$$

$$\begin{cases} 9x^2 - y^2 = 143 \end{cases}$$

$$\begin{cases} 3x - y = 13 \end{cases}$$

$$\begin{cases} (3x + y)(3x - y) = 143 \end{cases}$$

$$\begin{cases} 3x - y = 13 \end{cases}$$

$$\begin{cases} (3x + y) \cdot 13 = 143; 3x + y = 11 \end{cases}$$

$$\begin{cases} 3x - y = 13 \end{cases}$$

$$\begin{cases} 3x + y = 11 \end{cases}$$

$$(1) + (2) \begin{cases} 6x = 24; x = 4 \end{cases}$$

$$(1) - (2) \begin{cases} -2y = 2; y = -1 \end{cases}$$

37)

$$\begin{cases} x^2 + x^2 y = 2 \end{cases}$$

$$\begin{cases} x^2 - xy^2 = 2 \end{cases}$$

$$(1) - (2) \begin{cases} x^2 y + xy^2 = 0 \quad xy(x + y) = 0 \quad x = 0 \vee y = 0 \vee y = -x \end{cases}$$

$$(1) \begin{cases} x^2 + x^2 y = 2 \end{cases}$$

$$\begin{cases} x = 0 \text{ imposs.} \\ 0 = 2 \end{cases} \vee \begin{cases} y = 0 \\ x^2 = 2 \quad x = \pm\sqrt{2} \end{cases} \vee \begin{cases} y = -x \\ x^2 - x^3 = 2 \\ x^3 - x^2 + 2 = 0 \end{cases}$$

Ruffini

$$(x+1)(x^2 - 2x + 2) = 0$$

$$x = -1$$

$$x^2 - 2x + 2 = 0 \text{ imposs. } (\Delta < 0)$$

$$\begin{cases} x = -1 \end{cases}$$

$$\begin{cases} y = -x = 1 \end{cases}$$

$$\begin{cases} x = \sqrt{2} \\ y = 0 \end{cases} \begin{cases} x = -\sqrt{2} \\ y = 0 \end{cases} \begin{cases} x = -1 \\ y = 1 \end{cases}$$

39)

$$\begin{cases} x^2 + 2y^2 + 3z^2 = 9 \\ 3y^2 - 2z^2 = 1 \\ 3x^2 + 4y^2 = 16 \end{cases}$$

$$x^2 = X, y^2 = Y, z^2 = Z$$

$$\begin{cases} X + 2Y + 3Z = 9 \\ 3Y - 2Z = 1 \\ 3X + 4Z = 16 \end{cases}$$

$$\begin{cases} Y = \frac{2Z+1}{3} \\ X = \frac{16-4Z}{3} \\ \frac{16-4Z}{3} + 2 \cdot \frac{2Z+1}{3} + 3Z = 9 \end{cases}$$

$$16 - 4Z + 4Z + 2 + 9Z = 27$$

$$9Z = 9$$

$$Z = 1$$

$$\begin{cases} X = \frac{16-4Z}{3} = \frac{16-4}{3} = \frac{12}{3} = 4 \\ Y = \frac{2Z+1}{3} = \frac{3}{3} = 1 \\ Z = 1 \end{cases}$$

$$x^2 = 4 \rightarrow x = \pm 2$$

$$y^2 = 1 \rightarrow y = \pm 1$$

$$z^2 = 1 \rightarrow z = \pm 1$$