

**CORREZIONE SISTEMI RISOLUBILI CON ARTIFICI**

4)

$$\begin{cases} x^2 + y^2 + z^2 = 11 \\ xy + xz + yz = -1 \\ x - y + z = 5 \end{cases}$$

$$\begin{cases} (x + y + z)^2 - 2xy - 2xz - 2yz = 11 \\ xy + xz + yz = -1 \\ x - y + z = 5 \end{cases}$$

$$\begin{cases} (x + y + z)^2 - 2(xy + xz + yz) = 11 \\ xy + xz + yz = -1 \\ x - y + z = 5 \end{cases}$$

$$\begin{cases} (x + y + z)^2 - 2 \cdot (-1) = 11; & (x + y + z)^2 = 9; & x + y + z = \pm 3 \\ xy + xz + yz = -1 \\ x - y + z = 5 \end{cases}$$

$$\begin{cases} x + y + z = 3 \\ xy + xz + yz = -1 \\ x - y + z = 5 \end{cases}$$

$$\begin{cases} x + y + z = -3 \\ xy + xz + yz = -1 \\ x - y + z = 5 \end{cases}$$

$$(1) - (3) \begin{cases} 2y = -2; & y = -1 \\ (1) \quad \begin{cases} x + y + z = 3 \\ (2) \quad \begin{cases} xy + xz + yz = -1 \end{cases} \end{cases} \end{cases}$$

$$(1) - (3) \begin{cases} 2y = -8; & y = -4 \\ (1) \quad \begin{cases} x + y + z = -3 \\ (2) \quad \begin{cases} xy + xz + yz = -1 \end{cases} \end{cases} \end{cases}$$

$$\begin{cases} y = -1 \\ x - 1 + z = 3; & x + z = 4 \\ -x + xz - z = -1; & x + z - xz = 1 \end{cases}$$

$$\begin{cases} y = -4 \\ x - 4 + z = -3; & x + z = 1 \\ -4x + xz - 4z = -1; & 4x + 4z - xz = 1; & 4(x + z) - xz = 1 \end{cases}$$

$$\begin{cases} y = -1 \\ x + z = 4 \\ 4 - xz = 1; & xz = 3 \end{cases}$$

$$\begin{cases} y = -4 \\ x + z = 1 \\ 4 - xz = 1; & xz = 3 \end{cases}$$

$$\boxed{\begin{cases} y = -1 \\ x = 1 \\ z = 3 \end{cases}} \quad \boxed{\begin{cases} y = -1 \\ x = 3 \\ z = 1 \end{cases}}$$

$$t^2 - t + 3 = 0 \quad \text{impossibile in } \mathbb{R}$$

5)

$$\begin{cases} x - y + z = a \\ x - 3y + 2z + 1 = 0 \\ x^2 + z^2 = 2(y^2 + 1) \end{cases}$$

$$\begin{cases} x = a + y - z \\ a + y - z - 3y + 2z + 1 = 0 \\ x^2 + z^2 = 2y^2 + 2 \end{cases}$$

$$\begin{cases} x = a + y - z \\ -2y + z = -1 - a; \quad z = 2y - 1 - a \\ x^2 + z^2 = 2y^2 + 2 \end{cases}$$

$$\begin{cases} z = 2y - 1 - a \\ x = a + y - z = a + y - (2y - 1 - a) = a + y - 2y + 1 + a = 2a + 1 - y \\ x^2 + z^2 = 2y^2 + 2 \end{cases}$$

$$\begin{cases} x = 2a + 1 - y \\ z = 2y - 1 - a \\ (2a + 1 - y)^2 + (2y - 1 - a)^2 = 2y^2 + 2 \end{cases}$$

$$\begin{cases} x = 2a + 1 - y \\ z = 2y - 1 - a \\ 4a^2 \cancel{1} + y^2 + 4a - 4ay - 2y + 4y^2 \cancel{1} + a^2 - 4y - 4ay + 2a = 2y^2 \cancel{2} \end{cases}$$

$$\begin{cases} x = 2a + 1 - y \\ z = 2y - 1 - a \\ 3y^2 - 8ay - 6y + 5a^2 + 6a = 0; \quad 3y^2 - 2(4a + 3)y + a(5a + 6) = 0 \end{cases}$$

$$\begin{cases} x = 2a + 1 - y \\ z = 2y - 1 - a \\ y_{1,2} = \frac{4a + 3 \pm \sqrt{16a^2 + 24a + 9 - 15a^2 - 18a}}{3} = \frac{4a + 3 \pm \sqrt{a^2 + 6a + 9}}{3} = \\ = \frac{4a + 3 \pm (a + 3)}{3} = \begin{cases} \frac{4a \cancel{3} - a \cancel{3}}{3} = \frac{3a}{3} = a \\ \frac{4a + 3 + a + 3}{3} = \frac{5a + 6}{3} \end{cases} \end{cases}$$

$$\begin{cases} \boxed{y = a} \\ \boxed{x} = 2a + 1 - y = 2a + 1 - a = \boxed{a + 1} \\ \boxed{z} = 2y - 1 - a = 2a - 1 - a = \boxed{a - 1} \end{cases} \quad \begin{cases} \boxed{y} = \frac{5a + 6}{3} \\ \boxed{x} = 2a + 1 - y = 2a + 1 - \frac{5a + 6}{3} = \frac{6a + 3 - 5a - 6}{3} = \frac{a - 3}{3} \\ \boxed{z} = 2y - 1 - a = 2 \cdot \frac{5a + 6}{3} - 1 - a = \frac{10a + 12 - 3 - 3a}{3} = \frac{7a + 9}{3} \end{cases}$$

6)

$$\begin{cases} x^2y + xy^2 = 0 \\ x + y + z = 1 \\ x^2 + y^2 + z^2 = 2 \end{cases}$$

$$\begin{cases} xy(x+y) = 0; \quad x=0 \vee y=0 \vee x+y=0 \\ x + y + z = 1 \\ x^2 + y^2 + z^2 = 2 \end{cases}$$

$$\begin{cases} x=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases} \vee \begin{cases} y=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases} \vee \begin{cases} x+y=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases}$$

$$\begin{cases} x=0 \\ y+z=1 \\ y^2+z^2=2 \end{cases} \vee \begin{cases} y=0 \\ x+z=1 \\ x^2+z^2=2 \end{cases} \vee \begin{cases} x+y=0 \\ z=1 \\ x^2+y^2+1=2; \quad x^2+y^2=1 \end{cases}$$

$$\begin{cases} x=0 \\ y=1-z \\ (1-z)^2+z^2=2 \end{cases} \vee \begin{cases} y=0 \\ x=1-z \\ (1-z)^2+z^2=2 \end{cases} \vee \begin{cases} x+y=0 \\ z=1 \\ (x+y)^2-2xy=1; \quad -2xy=1; \quad xy=-1/2 \end{cases}$$

$$\begin{cases} x=0 \\ y=1-z \\ z=\frac{1\pm\sqrt{3}}{2} \end{cases} \vee \begin{cases} y=0 \\ x=1-z \\ z=\frac{1\pm\sqrt{3}}{2} \end{cases} \vee \begin{cases} z=1 \\ y=-x \\ -x^2=-\frac{1}{2}; \quad x^2=\frac{1}{2}; \quad x=\pm\sqrt{\frac{1}{2}}=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2} \end{cases}$$

$$\begin{cases} x=0 \\ y=\frac{1+\sqrt{3}}{2} \\ z=\frac{1-\sqrt{3}}{2} \end{cases}, \begin{cases} x=0 \\ y=\frac{1-\sqrt{3}}{2} \\ z=\frac{1+\sqrt{3}}{2} \end{cases}, \begin{cases} y=0 \\ x=\frac{1+\sqrt{3}}{2} \\ z=\frac{1-\sqrt{3}}{2} \end{cases}, \begin{cases} y=0 \\ x=\frac{1-\sqrt{3}}{2} \\ z=\frac{1+\sqrt{3}}{2} \end{cases}, \begin{cases} z=1 \\ y=\frac{\sqrt{2}}{2} \\ x=-\frac{\sqrt{2}}{2} \end{cases}, \begin{cases} z=1 \\ y=-\frac{\sqrt{2}}{2} \\ x=\frac{\sqrt{2}}{2} \end{cases}$$

Ricapitolando, le soluzioni sono le 6 terne :

$$\left(0, \frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right) \quad \left(0, \frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$$

$$\left(\frac{1+\sqrt{3}}{2}, 0, \frac{1-\sqrt{3}}{2}\right) \quad \left(\frac{1-\sqrt{3}}{2}, 0, \frac{1+\sqrt{3}}{2}\right)$$

$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right) \quad \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1\right)$$

7)

$$\begin{cases} x + y - 6z = 0 \\ x^2 + y^2 + 36z^2 = 56 \\ xyz = 8 \end{cases}$$

$$\begin{cases} x + y = 6z \\ xy = \frac{8}{z} \quad (z \neq 0) \\ (x + y)^2 - 2xy + 36z^2 = 56 \end{cases}$$

$$\begin{cases} x + y = 6z \\ xy = \frac{8}{z} \\ 36z^2 - \frac{16}{z} + 36z^2 = 56 \end{cases}$$

$$72z^2 - \frac{16}{z} = 56; \quad 72z^3 - 16 = 56z; \quad 9z^3 - 2 = 7z;$$

$$9z^3 - 7z - 2 = 0$$

$$P(1) = 0$$

$$\begin{array}{c|ccc|c} & 9 & 0 & -7 & -2 \\ 1 & & 9 & 9 & 2 \\ \hline & 9 & 9 & 2 & 0 \end{array}$$

$$(z-1)(9z^2 + 9z + 2) = 0$$

$$z = 1; \quad z = \frac{-9 \pm \sqrt{81 - 72}}{18} = \frac{-9 \pm \sqrt{9}}{18} = \frac{-9 \pm 3}{18} = \begin{cases} -\frac{12}{18} = -\frac{2}{3} \\ -\frac{6}{18} = -\frac{1}{3} \end{cases}$$

$$\begin{cases} z = 1 \\ x + y = 6z = 6 \cdot 1 = 6 \\ xy = \frac{8}{z} = \frac{8}{1} = 8 \end{cases} \quad \begin{cases} z = -\frac{2}{3} \\ x + y = 6z = 6 \cdot \left(-\frac{2}{3}\right) = -4 \\ xy = \frac{8}{z} = \frac{8}{-\frac{2}{3}} = -12 \end{cases} \quad \begin{cases} z = -\frac{1}{3} \\ x + y = 6z = 6 \cdot \left(-\frac{1}{3}\right) = -2 \\ xy = \frac{8}{z} = \frac{8}{-\frac{1}{3}} = -24 \end{cases}$$

$$\boxed{\begin{cases} z = 1 \\ x = 2 \\ y = 4 \end{cases}} \quad \boxed{\begin{cases} z = 1 \\ x = 4 \\ y = 2 \end{cases}}$$

$$\boxed{\begin{cases} z = -\frac{2}{3} \\ x = -6 \\ y = 2 \end{cases}} \quad \boxed{\begin{cases} z = -\frac{2}{3} \\ x = 2 \\ y = -6 \end{cases}}$$

$$\boxed{\begin{cases} z = -\frac{1}{3} \\ x = -6 \\ y = 4 \end{cases}} \quad \boxed{\begin{cases} z = -\frac{1}{3} \\ x = 4 \\ y = -6 \end{cases}}$$