

## SISTEMI SIMMETRICI

Correzione degli esercizi n. 5, 9, 11, 13, 15, 16

5)

$$\begin{cases} x + y = \frac{1}{6} \\ x^2 + y^2 = \frac{13}{36} \end{cases}$$

$$\begin{cases} x + y = \frac{1}{6} \\ (x + y)^2 - 2xy = \frac{13}{36} \end{cases} \quad \begin{cases} x + y = \frac{1}{6} \\ \frac{1}{36} - 2xy = \frac{13}{36}; \quad -2xy = \frac{12}{36}; \quad xy = -\frac{1}{6} \end{cases}$$

$$t^2 - \frac{1}{6}t - \frac{1}{6} = 0$$

$$6t^2 - t - 1 = 0$$

$$t_{1,2} = \frac{1 \pm \sqrt{1+24}}{12} = \frac{1 \pm 5}{12} = \begin{cases} -\frac{4}{12} = -\frac{1}{3} \\ \frac{6}{12} = \frac{1}{2} \end{cases}$$

$$\begin{cases} x = -\frac{1}{3} \\ y = \frac{1}{2} \end{cases} \quad \vee \quad \begin{cases} x = \frac{1}{2} \\ y = -\frac{1}{3} \end{cases}$$

9)

$$\begin{cases} xy = 16 \\ x^2 + y^2 = 32 \end{cases}$$

$$\begin{cases} xy = 16 \\ (x + y)^2 - 2xy = 32 \end{cases}$$

$$\begin{cases} xy = 16 \\ (x + y)^2 - 32 = 32; \quad (x + y)^2 = 64; \quad x + y = \pm 8 \end{cases}$$

$$\begin{cases} xy = 16 \\ x + y = 8 \end{cases} \quad \vee \quad \begin{cases} xy = 16 \\ x + y = -8 \end{cases}$$

$$t^2 - 8t + 16 = 0$$

$$t^2 + 8t + 16 = 0$$

$$(t - 4)^2 = 0$$

$$(t + 4)^2 = 0$$

$$t_{1,2} = 4$$

$$t_{1,2} = -4$$

$$\begin{cases} x = 4 \\ y = 4 \end{cases} \text{ soluz. "doppia"}$$

$$\begin{cases} x = -4 \\ y = -4 \end{cases} \text{ soluz. "doppia"}$$

11)

$$\begin{cases} xy = 3 \\ x^2 + y^2 = 14 \end{cases} \quad \begin{cases} xy = 3 \\ (x+y)^2 - 2xy = 14 \end{cases}$$

$$\begin{cases} xy = 3 \\ (x+y)^2 - 6 = 14; \quad (x+y)^2 = 20; \quad x+y = \pm\sqrt{20} = \pm 2\sqrt{5} \end{cases}$$

$$\begin{cases} x+y = 2\sqrt{5} \\ xy = 3 \end{cases} \quad \begin{cases} x+y = -2\sqrt{5} \\ xy = 3 \end{cases}$$

$$t^2 - 2t\sqrt{5} + 3 = 0 \quad t^2 + 2t\sqrt{5} + 3 = 0$$

$$t_{1,2} = \sqrt{5} \pm \sqrt{5-3} = \sqrt{5} \pm \sqrt{2} \quad t_{1,2} = -\sqrt{5} \pm \sqrt{5-3} = -\sqrt{5} \pm \sqrt{2}$$

$$\begin{cases} x = \sqrt{5} + \sqrt{2} \\ y = \sqrt{5} - \sqrt{2} \end{cases} \quad \begin{cases} x = \sqrt{5} - \sqrt{2} \\ y = \sqrt{5} + \sqrt{2} \end{cases} \quad \begin{cases} x = -\sqrt{5} - \sqrt{2} \\ y = -\sqrt{5} + \sqrt{2} \end{cases} \quad \begin{cases} x = -\sqrt{5} + \sqrt{2} \\ y = -\sqrt{5} - \sqrt{2} \end{cases}$$

13)

$$\begin{cases} x+y = 2a \\ x^2 + y^2 = 2(a^2 + b^2) \end{cases}$$

$$\begin{cases} x+y = 2a \\ (x+y)^2 - 2xy = 2(a^2 + b^2); \end{cases}$$

$$4a^2 - 2xy = 2a^2 + 2b^2; \quad -2xy = -2a^2 + 2b^2;$$

$$2xy = 2a^2 - 2b^2; \quad xy = a^2 - b^2$$

$$\begin{cases} x+y = 2a \\ xy = a^2 - b^2 \end{cases}$$

$$t^2 - 2at + (a^2 - b^2) = 0$$

$$t = a \pm \sqrt{a^2 - (a^2 - b^2)} = a \pm \sqrt{\cancel{a^2} - \cancel{a^2} + b^2} = a \pm b$$

$$\begin{cases} x = a+b \\ y = a-b \end{cases} \quad \vee \quad \begin{cases} x = a-b \\ y = a+b \end{cases}$$

15)

$$\begin{cases} x^3 + y^3 = 14 \\ x + y = 2 \end{cases} \quad \boxed{x^3 + y^3 = (x + y)^3 - 3x^2y - 3xy^2 = (x + y)^3 - 3xy(x + y)}$$

$$\begin{cases} (x + y)^3 - 3xy(x + y) = 14 \\ x + y = 2 \end{cases}$$

$$\begin{cases} 8 - 6xy = 14; & -6xy = 6; & xy = -1 \\ x + y = 2 \end{cases}$$

$$t^2 - 2t - 1 = 0$$

$$t_{1,2} = 1 \pm \sqrt{1+1} = 1 \pm \sqrt{2}$$

$$\begin{cases} x = 1 - \sqrt{2} \\ y = 1 + \sqrt{2} \end{cases} \vee \begin{cases} x = 1 + \sqrt{2} \\ y = 1 - \sqrt{2} \end{cases}$$

16)

$$\begin{cases} x^3 + y^3 = \frac{1}{4} \\ x + y = 1 \end{cases}$$

$$\begin{cases} (x + y)^3 - 3xy(x + y) = \frac{1}{4} \\ x + y = 1 \end{cases}$$

$$\begin{cases} 1 - 3xy = \frac{1}{4}; & 3xy = 1 - \frac{1}{4}; & 3xy = \frac{3}{4}; & xy = \frac{1}{4} \\ x + y = 1 \end{cases}$$

$$t^2 - t + \frac{1}{4} = 0$$

$$4t^2 - 4t + 1 = 0$$

$$(2t - 1)^2 = 0$$

$$t_{1,2} = \frac{1}{2}$$

$$\begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} \end{cases}$$

