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Fundamental Theorem of Algebra

The fundamental theorem of algebra (FTA) states:
"Every polynomial of degree *n* with complex coefficients has *n* roots in the complex numbers".

Early work with equations only considered positive real **roots** so the FTA was not relevant.

Cardan realized that one could work with numbers outside of the reals while studying a formula for the roots of a cubic equation. While solving $x^3 = 15x + 4$ using the formula he got an answer involving the square root of -121. He manipulated this to obtain the correct answer, $x = 4$, even though he did not understand exactly what he was doing with these "complex numbers" .

In 1572 **Bombelli** created rules for these "complex numbers" .

In 1637 **Descartes** said that one can "imagine" for every equation of degree *n*, *n* roots, but these imagined roots do not correspond to any real quantity.

Albert Girard, a Flemish mathematician, was the first to claim that there are always *n* solutions to a polynomial of degree *n* in 1629 in "L'invention en algèbre". He does not say that the solutions are of the form a + b*i*, a, b real. Many mathematicians accepted Girard's claim that a polynomial equation must have *n* roots, and proceeded to try to show that these roots were of the form a + b*i*, a, b real, instead of first showing that they actually existed.

This was the *downfall* of many attempts to prove the FTA. Various people tried to disprove and prove the FTA.

Leibniz gave a "proof" that the FTA was false in 1702 by saying $x^4 + t^4$ couldn't be written as the product of 2 real quadratic factors. He did not realize that the square root of *i* could be written in the form $a + bi$, a, b real. In 1742 **Euler** showed Leibniz's *counterexample* was incorrect.

D'Alembert in 1746 and Euler in 1749 attempted proofs of the FTA. **Lagrange** and **Laplace** attempted proofs, but also failed, as they were assuming the existence of roots.

Gauss is credited with the first proof of the FTA in his doctoral thesis of 1799. He *spotted* the error in the others' proofs, that they were assuming the roots existed and then trying to deduce properties of them. This proof has some gaps in it and is not considered rigorous by today's standards.

In 1814 the Swiss accountant **Jean Robert Argand** published a proof of the FTA and two years later Gauss published a second proof.

This time Gauss's proof was complete and correct. He did a third proof the same year, and in 1849 a fourth proof was completed.

Root = "radice" (anche in botanica!)
In matematica, "root" può significare
- "radice" nel senso di "radice quadrata, cubica …"

o anche, **parlando di equazioni**,
- "radice" nel senso di "**soluzione**"

e, **parlando di polinomi**,
- "radice" nel senso di "**zero**"

Girolamo **Cardano**, italiano (1501-1576)

Rafael **Bombelli**, italiano (1526-1572)

René **Descartes o "Cartesio"**, filosofo e matematico francese (1596-1650)

Albert **Girard**, fiammingo (1595-1632)

downfall = caduta, crollo

Gottfried **Leibniz,** filosofo e matematico tedesco (1646-1716)
Leonhard **Euler (Euléro)**, svizzero (1707-1783)

counterexample = controesempio

Jean **d'Alembert** (1717-1783) filosofo, scienziato e matematico francese

Joseph-Louis **Lagrange** (1736-1813) scienziato e matematico franco-italiano

Pierre-Simon de **Laplace** (1749-1827) scienziato e matematico francese

Carl Friedrich **Gauss**, tedesco (1777-1855)

to spot = (in questo caso) individuare, riconoscere, scoprire

Jean Robert **Argand**, svizzero (1768-1822)

Queste brevi note storiche sul Teorema Fondamentale dell'Algebra ci hanno portato, in particolare, ad incontrare
- alcuni fra i più importanti **filosofi** di tutti i tempi come Cartesio e Leibniz,
- alcuni fra i sommi **matematici** di tutti i tempi come Eulero e Gauss,
- e alcuni fra i massimi **protagonisti della cultura e della scienza** come D'Alembert, Lagrange e Laplace.